INSTITUTE OF ENGINEERING

Model Entrance Exam

(Set-1 Solutions)

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Date: 2080/02/13
(May-27)

Duration: 2 hours
Time: 7 AM – 9 AM

Section-A (1 marks)

1) a
2) d
3) a
4) a
5) d
6) c
7) d
8) c
9) c
10) b
11) a
12) a
13) b $\lim_{x\to 9}\frac{x^{\frac{3}{2}}-27}{x-9}$ $\left[\frac{0}{0}\ form\right]$

$= \lim_{x\to 9}\frac{\frac{3}{2}x^{\frac{1}{2}}-0}{1-0}$ $[L'hospitals\ rule]$

$= \frac{3}{2}\times(9)^{1/2} = \frac{3}{2}\times(3^2)^{\frac{1}{2}} = \frac{3}{2}\times 3 = \frac{9}{2}$

14) d $y = \tan^{-1}(\cot x) + \cot^{-1}(tanx) = \frac{\pi}{2} - \cot^{-1}(\cot x) + \frac{\pi}{2} - \tan^{-1}(\tan x) = \pi - 2x$

$\frac{dy}{dx} = -2$

15) b $f(x) = x^3 - 12x^2 + 45x$

$f'(x) = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3, 5$

$f(0) = 0, f(7) = 70, f(3) = 54, f(5) = 50$

∴ Greatest value = 70

16) b $\int\frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}dx = \int\frac{\sin x+\cos x}{\sqrt{sin^2x+cos^2x+2\sin x.\cos x}}dx = \int\frac{\sin x+\cos x}{\sqrt{(\sin x+\cos x)^2}}dx = \int dx = x + c$

17) a Sum of roots = 0 and product of roots = 0

$-\frac{b}{a} = 0$ and $\frac{c}{a} = 0$

$b = 0$ and $c = 0$

18) a Let a be the first term and l be the n^{th} term.

As given, T_n = l =128 and S_n = 255.

$S_n = \frac{lr-a}{r-1}$

$255 = \frac{2(128)-a}{2-1}$

$a = 1$

19) c Replace i by -i. required complex number = $\frac{1}{-i-1} = \frac{-1}{i+1}$

20) b If number of persons be n, then total number of handshakes is

${}^nC_2 = 66$

$n(n-1) = 132$

$(n+11)(n-12) = 0$

$n = 12$

21) d $|A| = -1 \neq 0$

∴ A is invertible matrix.

22) a $x^2 = 16$

$x = \pm 4$

$2x = 6$

$x = 3$

There is no value of x which satisfies both the above equations.

Thus, A = $\emptyset$

23) a For horizontal line, slope = 0

$\left(\frac{3+\lambda}{4-2\lambda}\right) = 0$

$\lambda = -3$

24) c The equation of tangent to the circle
$(x-h)^2+(y-k)^2=a^2$ is
$XX_1+YY_1=a^2$
where, $X=x-h, Y=y-k$
and $X_1=x_1-h, Y_1=y_1-k$
$(x-4)(2-4)+(y-7)(3-7)=20$
$-2(x-4)-4(y-7)-20=0$
$(x-4)+2(y-7)+10=0$
$x+2y-8=0$, whose slope $=-\frac{1}{2}$

25) a Given, $x=my+k$
$my=x-k$
$y=\frac{1}{m}x-\frac{k}{m}$
Using condition of tangency,
$-\frac{k}{m}=-a\left(\frac{1}{m}\right)^1$
$k=\frac{a}{m}$

26) d Given ellipse is:
$5x^2+9y^2=45$
$\frac{x^2}{9}+\frac{y^2}{5}=1$
Latus Rectum $=\frac{2b^2}{a}=\frac{2.5}{3}=\frac{10}{3}$

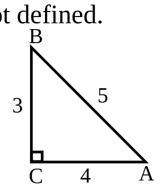
27) b $cos^2\alpha+cos^2\beta+cos^2\gamma=1$
$cos^2\alpha+cos^2\left(\frac{\pi}{2}-\alpha\right)+cos^2\gamma=1$
$cos^2\alpha+sin^2\alpha+cos^2\gamma=1$
$cos^2\gamma=0$
$\gamma=\frac{\pi}{2}$

28) a $\sin x+sin^2x=1$
$\sin x=1-sin^2x$
$sinx=cos^2x$
$cos^2x+cos^4x=cos^2x+(cos^2x)^2=cos^2x+(\sin x)^2=cos^2x+sin^2x=1$

29) b $\sin\theta-\cos\theta=0$
$\sin\theta=\cos\theta$
$\tan\theta=1=\tan^{-1}\left(\frac{\pi}{4}\right)$
$\theta=\frac{\pi}{4}$

30) a For $x=-\frac{1}{2}$, $\sin^{-1}\left(\frac{1}{x}\right)=\sin^{-1}(-2)$, which is not defined.

31) b $\sin A=\sqrt{1-cos^2A}=\frac{3}{5}$
$\sin B=\sqrt{1-cos^2B}=\frac{4}{5}$
$a:b:c=\sin A:\sin B:\sin C=\frac{3}{5}:\frac{4}{5}:1=3:4:5$



32) a $\frac{|\vec{a}\times\vec{b}|}{\vec{a}.\vec{b}}=\frac{ab\sin\theta}{ab\cos\theta}=\tan\theta$

33) b

34) a At highest point of motion, the acceleration of ball is equal to acceleration due to gravity, even though the ball is at rest.

35) a

36) d

37) b

38) b

39) b

40) c

41) c

42) a If liquid is heated from the top, heat energy will be transferred from top to the bottom through conduction while convection takes place from bottom to top.

43) d

44) a

45) d Work, W = $q(V_2-V_1)$. For equipotential surface: $V_1=V_2$
Therefore, W = 0

46) b

47) b
48) b
49) d Generally, magnetic lines of force due to earth's field are closed curves emerging from N-pole and ending to the S-pole. Lines of force due to earth's horizontal field are parallel and straight.
50) d Isobars have equal mass i.e., same number of nucleons (mass of proton + mass of neutron).
$_{18}Ar^{40}$, $_{19}K^{40}$, $_{20}Ca^{40+}$
51) c
52) c l=2 indicated d-subshell. It can accommodate 10 electrons.
53) a +7 -1 +3 0
$Cr_2O_7 + H^+ + I^- \longrightarrow Cr^{3+} + H_2O + I_2$
Here, 'Cr' is reduced and 'I' is oxidized.
54) a 'B' in BF_3 has electron deficient centre. So, it accepts electron i.e., electron pair acceptor – Lewis acid.
55) b Due to absence of d-orbital, Nitrogen cannot form pentahalide.
56) c
57) b Hydration energies of the cations of alkaline earth metals decreases as we go down the group because of increase in their ionic radii.
58) a
59) b
60) d In Wurtz reaction, two molecules of alkyl halide react with 2 Na-atoms to give the hydrocarbon with double C-atom.

$$\text{R}-\boxed{\text{Cl} + 2\text{Na} + \text{Cl} - \text{R}} \xrightarrow{\text{ether}} \text{R} - \text{R} + 2\text{NaCl}$$

As methane contains only one C-atom, it cannot be synthesized by this reaction.

<u>Section-B</u> (2 marks)

61) d
62) c
63) a
64) a
65) c $\lim_{x\to 0} \frac{\sin x + \log(1-x)}{x^2}$ $\left[\frac{0}{0}\ form\right]$

$= \lim_{x\to 0} \frac{\cos x - \frac{1}{1-x}}{2x}$ $\left[\frac{0}{0}\ form\right]$

$= \lim_{x\to 0} \frac{-\sin x - \frac{1}{(1-x)^2}}{2} = \frac{-0-1}{2} = -\frac{1}{2}$

66) c $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$

$\sin^{-1} y = \frac{\pi}{2} - \sin^{-1} x$

$\sin^{-1} y = \cos^{-1} x$

$y = \sin(\cos^{-1} x)$

Put $\cos^{-1} x = \theta$

$\therefore \cos\theta = x$

$y = \sin\theta = \sqrt{1 - cos^2\theta} = \sqrt{1 - x^2}$

$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}(-2x) = \frac{-x}{\sqrt{1-x^2}} = -\frac{x}{y}$

67) b As given, $\frac{d}{d\theta}(\sin\theta) = k$

$\cos\theta = k$

$\frac{d}{d\theta}(\tan\theta) = sec^2\theta = \frac{1}{k^2}$

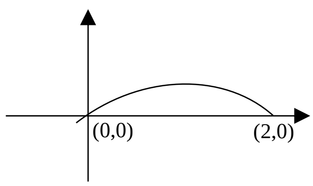
68) c $\int \frac{\cos 2x - 1}{\cos 2x + 1} dx = \int \frac{-2sin^2 x}{2cos^2 x} dx = \int tan^2 x\ dx = \int (sec^2 x - 1)\, dx = -\tan x + x + c = x - \tan x + c$

69) b Given curve, $y = 2x - x^2$

$(x-1)^2 = -(y-1)$

The curve cuts x-axis at (0,0) and (2,0)

$A = \int_0^2 (2x - x^2)\, dx = \left[x^2 - \frac{x^3}{3}\right]_0^2 = \frac{4}{3}$



70) d $T_3 = T_{2+1} = {}^nC_2\,(x^2)^{n-2}\left(-\frac{1}{x^3}\right)^2 = {}^nC_2\,x^{2n-10}$
This does not contain x if
$2n - 10 = 0$
$n = 5$

71) a $a + ar + ar^2 + \cdots = 3$
$\frac{a}{1-r} = 3$
$a = 3(1-r)$ …(1)
$a^2 + a^2r^2 + a^2r^4 + \cdots = \frac{9}{2}$
$\frac{a^2}{1-r^2} = \frac{9}{2}$
$\frac{9(1-r)^2}{1-r^2} = \frac{9}{2}$
$\frac{(1-r)(1-r)}{(1+r)(1-r)} = \frac{1}{2}$
$\frac{1-r}{1+r} = \frac{1}{2}$
On solving, $r = \frac{1}{3}$
From (1), $a = 3\left(1 - \frac{1}{3}\right) = 2$
Thus, Required sum $= \frac{a^3}{1-r^3} = \frac{8}{1-\frac{1}{27}} = \frac{108}{13}$

72) a Operate $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$
$\begin{vmatrix} x+1 & x+2 & x+4 \\ 2 & 3 & 4 \\ 6 & 8 & 10 \end{vmatrix}$
Operate $C_3 \to C_3 - C_2, C_2 \to C_2 - C_1$
$\begin{vmatrix} x+1 & 1 & 3 \\ 2 & 1 & 2 \\ 6 & 2 & 4 \end{vmatrix}$
Operate $R_3 \to R_3 - 2R_2$
$\begin{vmatrix} x+1 & 1 & 3 \\ 2 & 2 & 2 \\ 2 & 0 & 0 \end{vmatrix} = 2\begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} = -2$

73) d Let $y = f(x) = \frac{x}{1-x}$
$y - xy = x$
$x = \frac{y}{1+y}$
Which is defined for all $y \in R$ except $y = -1$
$\therefore R_f = (-\infty, -1) \cup (-1, \infty)$

74) b Given equation can be written as,
$x^2 + 2(1)xy + y^2 + 2(-4a)x + 2(-4a)y - 9a^2 = 0$
Required distance $= 2\sqrt{\frac{g^2 - ac}{a(a+b)}} = 2\sqrt{\frac{(-4a)^2 + 9a^2}{1(1+1)}} = 2\sqrt{\frac{25a^2}{2}} = 5\sqrt{2}a$

75) a The equation of circle through points of intersection of given lines and the coordinate axes is given by:
$(\lambda x - y + 1)(x - 2y + 3) + kxy = 0$
This represents a circle if
Coeff. of x^2 = Coeff. of y^2
i.e., $\lambda = (-1)(-2) = 2$

76) a As given, $\frac{2b^2}{a} = 8$
And $\frac{3}{\sqrt{5}} = \sqrt{1 + \frac{b^2}{a^2}}$
$\frac{9}{5} - 1 = \frac{b^2}{a^2}$
$\frac{4}{5} = \frac{2b^2}{a} \cdot \frac{1}{2a}$
$\frac{4}{5} = \frac{8.1}{2a}$
$a = 5$
And, $\frac{2b^2}{5} = 8$
$b^2 = 20$
So, Required equation is:

$\frac{x^2}{25} - \frac{y^2}{20} = 1$
$4x^2 - 5y^2 = 100$

77) a The equation of plane parallel to the given plane is $x - 2y + 2z = k$
As given, $\left|\frac{1-4+6-k}{\sqrt{1+4+4}}\right| = 1$
$|3 - k| = 3$
$k = 0,6$
$\therefore x - 2y + 2z = 6$ is the required plane.

78) b $\sin^2\theta - 2\cos\theta + \frac{1}{4} = 0$
$4\sin^2\theta - 8\cos\theta + 1 = 0$
$4 - 4cos^2\theta - 8\cos\theta + 1 = 0$
$4cos^2\theta + 8\cos\theta - 5 = 0$
$(2\cos\theta - 1)(2\cos\theta + 5) = 0$
$\cos\theta = \frac{1}{2}$, $\cos\theta = -\frac{5}{2}$ (not possible)
$\therefore \theta = \frac{\pi}{3}$

79) c $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$
$\frac{\cos A}{2RsinA} = \frac{\cos B}{2RsinB} = \frac{cosC}{2RsinC}$
$\cot A = \cot B = \cot C$
$A = B = C$
i.e., triangle is equilateral
$\therefore$ Area $= \frac{\sqrt{3}}{4} \times (2)^2 = \sqrt{3}$

80) b $v^2 = u^2 - 2as \Rightarrow 0 = u^2 - 2gh$
$\Rightarrow h = \frac{u^2}{2g}$
i.e., $h \propto u^2$
When velocity is doubled, maximum height becomes 4 times.
$\therefore h' = 4(50) = 200\ m$

81) b $T = \frac{2usin\theta}{g}$
$\Rightarrow \frac{T_1}{T_2} = \frac{sin\theta_1}{sin\theta_2} = \frac{sin\theta}{\sin(90° - \theta)} = tan\theta : 1$

82) b The inclination of person from vertical,
$tan\theta = \frac{v^2}{rg} = \frac{(10)^2}{50\times10} = \frac{1}{5}$
$\therefore \theta = \tan^{-1}\left(\frac{1}{5}\right)$

83) d K.E. of rotation = $\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 = \frac{1}{5}MR^2\omega^2 = \frac{1}{5}\times 1 \times (3\times10^{-2})^2 \times (50)^2 = 0.45\ J$

84) c Escape velocity for earth,
$v_e = \sqrt{\frac{2GM_e}{R_e}}$ ------- (i)
Escape velocity for moon,
$v_m = \sqrt{\frac{2GM_m}{R_m}}$ ------ (ii)
Dividing (ii) by (i), we get,
$\frac{v_m}{v_e} = \sqrt{\frac{M_m}{M_e}.\frac{R_e}{R_m}} = \sqrt{\frac{1}{81}\times 4}$
$\therefore v_m = \frac{2}{9}\times 11.2 = 2.5\ km/s$

85) c Weight of sphere = Weight of liquid displaced
$V\rho g = \frac{V}{3}\times 13.5g + \frac{2V}{3}\times 1.2g$
$\therefore \rho = \frac{13.5+2.4}{3} = 5.3$

86) a $l_t = l_0(1+\alpha t) \Rightarrow \alpha = \frac{l_t - l_0}{l_0 t} = \frac{\Delta l}{l_0 t} = \frac{0.08\times10^{-3}}{10\times10^{-3}\times100} = 8\times10^{-5}/°C$
$V_t = V_0(1+3\alpha t) = 1000(1+3\times8\times10^{-5}\times100) = 1002.4\ cc$

87) b $PV = \frac{m}{M}RT$
$\Rightarrow V = \frac{mRT}{MP} = \frac{2.2\times10^{-3}\times8.3\times10^3\times273}{44\times2\times1.01\times10^5} = 0.56\ litre$

88) d $n' = \frac{v}{v-v_s}.n$

$\Longrightarrow \frac{n'}{n} = \frac{v}{v-v_s} = \frac{v}{v-\frac{v}{10}} = \frac{10}{9}$

89) b $\mu = \frac{sini}{sinr}$

Given, i = 2r

$\therefore \mu = \frac{\sin(2r)}{sinr} = \frac{2sinr.cosr}{sinr} = 2cosr$

$\Longrightarrow cosr = \frac{\mu}{2} \Longrightarrow r = \cos^{-1}\frac{\mu}{2}$

$\therefore i = 2r = 2\cos^{-1}\frac{\mu}{2}$

90) b Focal length of lens = 1/P = 1/5 = 0.20 m = 20 cm

If R is the radius of curvature of each surface, then

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R}\right) = (\mu - 1)\frac{2}{R}$

$\therefore R = (\mu - 1).2f = (1.5 - 1) \times 2 \times 20 = 20\ cm$

91) a Effective Capacitance (C) = $2 + \frac{3\times6}{3+6} = 4\ \mu F$

$\therefore U = \frac{1}{2}CV^2 = \frac{1}{2} \times (4\mu F) \times (2V)^2 = 8\ \mu J$

92) d At resonance, the frequency of the supply power equals the natural frequency of the given LCR circuit.

Impedance of the circuit is given by,

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

At resonance,

$X_L = X_C$

$\therefore Z = R = 20\Omega$

Current (I) in the circuit = V/Z = 200/20 = 10 A

Hence, the average power transferred to the circuit in one complete cycle = VI

= 200 × 10

= 2000 W

93) a Min frequency = $\frac{hf - KE}{h}$ = 5.075×10^{14} Hz

94) d $CH_3CH_2CH_2OH \xrightarrow{conc.H_2SO_4, 170^\circ C} CH_3CH = CH_2 \xrightarrow{Br_2} CH_3 - CH(Br) - CH_2(Br) \xrightarrow{alc.KOH} CH_3 - C \equiv CH$

95) b

96) d From second law of Faraday,

$\frac{m_{Al}}{m_H} = \frac{E_{Al}}{E_H}$

$\frac{4.5}{m_H} = \frac{27/3}{1}$

$m_H = 0.5$ g

∵ Volume of 2g H_2 at STP = 22.4 L

∴ Volume of 0.5 g H_2 at STP = $\frac{22.4\times0.5}{2}$ L = 5.6 L

97) c No. of moles = 0.5/84

Molarity = $\frac{\text{No.of moles}}{\text{VOlume in litre}} = \frac{\frac{0.5}{84}}{250\times10^{-3}} = 0.024$ M

98) b H = 20%, C = (100–20)% = 80%

Atoms	%	Relative no. of atoms
C	80	$\frac{80}{12}$
H	20	$\frac{20}{1}$

i.e., C : H = $\frac{80}{12} : \frac{20}{1}$ = 1 : 3

So, empirical formula is CH_3 and its molecular formula will be $(CH_3)_n$, where n = 2, 3, 4, ….

i.e., C_2H_6

99) c Normality of 2M H_2SO_4 = 4N H_2SO_4

$V_aN_a + V_wN_w = (V_a + V_w)N_{mixture}$

or, 10×4 + 10×0 = (10+10) $N_{mixture}$

$N_{mixture}$ = 2N

Now, $V_{mixture} \times N_{mixture} = V_bN_b$
or, $10 \times 2 = V_b \times 2$
$\therefore V_b = 10$ mL

100) a Boiling point of HF is highest due to H-bonding. For other halogen acids, boiling point increase in the order $HCl < HBr < HI$. Therefore, most volatile (with lowest boiling point) is HCl.

Thank You!!!!!!