INSTITUTE OF ENGINEERING

Model Entrance Exam

(Set-2 Solutions)

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Date: 2080/02/27
(June-10)

Duration: 2 hours
Time: 8 AM – 10 AM

Section-A (1 marks)

1) a
2) b
3) a
4) a
5) c
6) d
7) d
8) a
9) c
10) a
11) c
12) b
13) c
14) c
15) a $F \propto \frac{1}{v}$, $F = \frac{C}{v}$
Where, c is a constant of proportionality.
$ma = \frac{C}{v}$
or, $m\frac{dv}{dt} = \frac{C}{v}$
or, $vdv = C\frac{dt}{m}$
Integrating both sides, we get,
$\frac{v^2}{2} = \frac{Ct}{m}$
or, $\frac{1}{2}mv^2 = Ct$
Kinetic energy, $K = \frac{1}{2}mv^2 = Ct$
or, $K \propto t$
16) b The escape velocity is independent of mass of the body and the direction of projection. It depends upon the gravitational potential at the point from where the body is launched. Since this potential slightly depends on the latitude and height of the point, the escape velocity depends on these factors.
17) c For a perfectly rigid body, both Young's modulus and bulk modulus is infinite.
18) b In SHM, total energy is: $E = \frac{1}{2}m\omega^2 A^2$
$E \propto A^2$
19) a
20) a Pressure, $P = \frac{1}{3}\frac{M}{V}{v_{rms}}^2 = \frac{2}{3}\frac{E}{V}$
Here, $E = \frac{1}{2}M{v_{rms}}^2$ is total energy of the gas.
At constant volume, $P \propto E$
21) c Sound waves are longitudinal waves that is why in air they cannot be polarized.
22) a The net charge enclosed by the sphere is zero.
23) b
24) d Above Curie temperature, ferromagnetic material become paramagnetic.
25) d When an ac voltage of 220 V is applied to a capacitor C, the charge on the plates is in phase with the applied voltage. As the circuit is pure capacitive so, the current developed leads the applied voltage by a phase angle of 90˚. Hence power delivered to the capacitor per cycle is:
$P = V_{rms}I_{rms}\cos 90^\circ = 0$
26) c
27) c Image formed is complete but has decreased intensity.
28) d Photoelectric current depends upon :
(i) the intensity of incident light
(ii) the potential difference applied between the two electrodes
(iii) the nature of the emitter material
29) b
30) b $r_1 = \frac{\Delta}{s-a}$
$s = \frac{13+14+15}{2} = 21$ and $\Delta = \sqrt{21(21-13)(21-14)(21-15)} = 84$

So, $r_1 = \frac{84}{21-13} = 10.5$

31) b Here, $A = \tan^{-1}x \Rightarrow \tan A = x$

$\sin 2A = \frac{2\tan A}{1+tan^2 A} = \frac{2x}{1+x^2}$

32) c $sin^2\theta = \frac{1}{4}$

$sin^2\theta = \left(\frac{1}{2}\right)^2$

$sin^2\theta = sin^2\frac{\pi}{6}$

$\theta = n\pi \pm \frac{\pi}{6}$

33) c Given expression is : $3\cos\theta + 4\sin\theta$

Here, $a = 3, b = 4$

The interval is : $\left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right] = [-5,5]$

34) a $\lim_{n\to\infty}\frac{1+2+3+\cdots+n}{n^2} = \lim_{n\to\infty}\frac{n(n+1)}{2n^2} = \lim_{n\to\infty}\frac{1}{2}\left(1+\frac{1}{n}\right) = \frac{1}{2}\left(1+\frac{1}{0}\right) = \frac{1}{2}$

35) b $\frac{d}{dx}\cos^{-1}(\sin x) = \frac{d\cos^{-1}(\sin x)}{d(sinx)}.\frac{d(sinx)}{dx} = -\frac{1}{\sqrt{1-sin^2x}}.\cos x = -\frac{\cos x}{\cos x} = -1$

36) d $\int_0^{\frac{1}{\sqrt{2}}}\frac{dx}{\sqrt{1-x^2}} = [\sin^{-1}x]_0^{\frac{1}{\sqrt{2}}} = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \sin^{-1}(0) = \frac{\pi}{4}$

37) c Area of equilateral triangle (A) = $\frac{\sqrt{3}}{4}a^2$

$\frac{dA}{dt} = \frac{\sqrt{3}}{4}.2a.\frac{da}{dt} = \frac{\sqrt{3}}{2}ak \quad \left[\because \frac{da}{dt} = k\ unit/sec\right]$

38) a $\sqrt{a+ib} + \sqrt{a-ib} = \sqrt{2|z|+2a}$

$\therefore \sqrt{7+24i} + \sqrt{7-24i} = \sqrt{2\times 25 + 2\times 7} = 8$

39) a Sum of the roots $(\alpha+\beta)$ = 3

$\frac{-(k-5)}{k-2} = 3$

$-k+5 = 3k-6$

$-4k = -11$

$k = \frac{11}{4}$

40) c $AB = 3I$

$A^{-1}(AB) = A^{-1}(3I)$

$IB = 3A^{-1}$

$A^{-1} = \frac{B}{3}$

41) b $\mathrm{A} - (\mathrm{B}\cap\mathrm{C}) = (A-B)\cup(A-C)$

42) a $y = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{2} + \cdots + \infty$

$y = log_e(1+x)$

$e^y = 1+x$

$x = e^y - 1$

43) c $f(x) = \frac{x}{2+x^2}$ is defined for any value of x in R.

So, Domain = (-∞, ∞)

44) a Projection of $\vec{b}$ on $\vec{a} = \frac{\vec{a}.\vec{b}}{|\vec{a}|} = \frac{5.2+(-3).1+1.2}{\sqrt{2^2+2^2+1}} = \frac{10-3+2}{3} = \frac{9}{3} = 3$

45) a Given, $a = 3b$

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{3b} + \frac{y}{b} = 1$

$x + 3y = 3b$ --- (1)

The point (1,2) lies in (1)

$1 + 6 = 3b$

$b = \frac{7}{3}$

So, the required equation is:

$x + 3y = 7$

46) c Centre of the circle $x^2 + y^2 - 6x - 4y - 3 = 0$ is (3,2)

Centre of concentric circle = (3,2)

Radius (r) = 5

The equation of the concentric circle is:

$(x-3)^2 + (y-2)^2 = 25$

$x^2 - 6x + 9 + y^2 - 4y + 4 = 25$

$x^2 + y^2 - 6x - 4y - 12 = 0$

47) b Equation of the parabola: $y^2 = 4ax$
It passes through (-3,2)
So, $4 = 4a(-3)$
$4a = -\frac{4}{3}$
Length of latus rectum (4a) = 4/3
(since, length cannot be negative)

48) d $x^2 - y^2 = 0$
$(x-y)(x+y) = 0$
$x - y = 0, x + y = 0$
It represents a pair of lines.

49) c $(a_1, b_1, c_1) = (1,2,3)$
$(a_2, b_2, c_2) = (3,-3,1)$
$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} = \frac{1.3+2(-3)+3.1}{\sqrt{1^2+2^2+3^2}\sqrt{3^2+(-3)^2+1^2}} = 0$
$\theta = 90^\circ$

50) d Positive ion is always smaller and negative ion is always larger than the parent atom. So, the correct order of size of iodine species will be: $I^- > I > I^+$.

51) c HC≡CH
(sp)

52) a Because of the highest electronegativity of Fluorine.

53) b CH_3COOK is a salt of strong base and weak acid. Its aqueous solution will be basic and pH value will be >7 ≈ 8.8.
Na_2CO_3 is a salt of strong base and weak acid. Its aqueous solution is also basic and pH value will be >10.
NH_4Cl is a salt of weak base and strong acid. So, its aqueous solution will be acidic and pH value will be less than 7.
$NaNO_3$ is the salt of strong acid and strong base. So, its aqueous solution is neutral and pH value will be equal to 7.

54) d Boiling point increases with increase in molecular mass. For the compounds with the same molecular mass, boiling point decreases with an increase in branching.

55) b Calgon is used for softening hard water.

56) c $x + 4(-2) = -3$
$x = -3 + 8 = +5$

57) b

58) d

59) d Normality = Basicity × Molarity = 2 × 0.3 = 0.6 N

60) c Atomic weight = N_A × mass of one atom =$6.022 \times 10^{23} \times 1.8 \times 10^{-22} = 108$

Section-B (2 marks)

61) d

62) c

63) a

64) c

65) b Here, $r = 12\ cm$, frequency, $v = \frac{7}{100} rps$
The angular speed of the insect is:
$\omega = 2\pi v = 2\pi \times \frac{7}{100} = 0.44\ rads^{-1}$
The linear speed of insect is: $v = \omega r = 0.44 \times 12 = 5.3\ cms^{-1}$

66) b Limiting friction, $f = \mu mg = 0.6 \times 1 \times 9.8 = 5.88\ N$
Applied force, $F = ma = 1 \times 5 = 5\ N$
As, $F < f$, so force of friction = 5 N

67) b According to the law of conservation of angular momentum, we get,
$L_i = L_f$
or, $I_i\omega_i = I_f\omega_f$
or, $\omega_f = \frac{I_i\omega_i}{I_f} = \left(\frac{I_i}{3I_i}\right)\omega_0 = \frac{\omega_0}{3}$

68) c $h = \frac{2S\cos\theta}{r\rho g}$
Mass of water in the first tube,
$m = \pi r^2 h\rho = \pi r^2 \times \left(\frac{2S\cos\theta}{r\rho g}\right) \times \rho = \frac{2\pi rS\cos\theta}{g}$
$m \propto r$

Hence, $\frac{m'}{m} = \frac{2r}{r} = 2$
or, $m' = 2m = 2 \times 5 = 10\ g$

69) c $T_i = 90^{\circ}C$ (Initial Temperature)
$T_f = 80^{\circ}C$ (Final Temperature)
$T_0 = 20^{\circ}C$ (Room Temperature)
Let time taken be t minutes.
According to Newton's law of cooling,
Rate of cooling $\frac{dT}{dt} = K\left[\frac{(T_i+T_f)}{2} - T_0\right]$
or, $\frac{90-80}{t} = K\left[\frac{(90+80)}{2} - 20\right]$
$K = \frac{10}{65\ t}$
In 2nd condition,
$T_i = 80^{\circ}C$ (Initial Temperature)
$T_f = 60^{\circ}C$ (Final Temperature)
Let time taken be t' minutes.
$\frac{80-60}{t'} = \frac{10}{65\ t}\left[\frac{(80+60)}{2} - 20\right]$
$\frac{20}{t'} = \frac{10}{65\ t}(50)$
$t' = \frac{13}{5}t$

70) d $P_1{V_1}^{\gamma} = P_2{V_2}^{\gamma}$
or, $PV^{\gamma} = P_2\left(\frac{V}{4}\right)^{\gamma}$
or, $P_2 = 4^{\gamma}P$
Now, for monatomic gases,
$\gamma = \frac{5}{3}$
$\therefore P_2 = 4^{5/3}$ P = 10.08 P

71) c Let L be the length of the pipe.
Fundamental frequency of closed pipe is:
$v_c = \frac{v}{4L}$
Where, v is the speed of sound in air.
Fundamental frequency of open pipe of same length is
$v_o = \frac{v}{2L}$
Divide (ii) by (i), we get,
$\frac{v_0}{v_c} = 2$
or, $v_0 = 2v_c = 2v$

72) c Initial energy of the combined system,
$U_1 = \frac{1}{2}C{V_1}^2 + \frac{1}{2}C{V_2}^2 = \frac{C}{2}\left({V_1}^2 + {V_2}^2\right)$
On joining the two condensers in parallel, common potential
$V = \frac{CV_1+CV_2}{C+C} = \frac{V_1+V_2}{2}$
Final energy of the combined system
$U_2 = \frac{1}{2}(C+C)\left(\frac{V_1+V_2}{2}\right)^2$
Decrease in energy
$\Delta U = U_1 - U_2 = \frac{1}{2}C\left({V_1}^2 + {V_2}^2\right) - \frac{1}{2}(2C)\left(\frac{V_1+V_2}{2}\right)^2 = \frac{C}{4}\left[2\left({V_1}^2 + {V_2}^2\right) - (V_1+V_2)^2\right] = \frac{C}{4}(V_1 - V_2)^2$

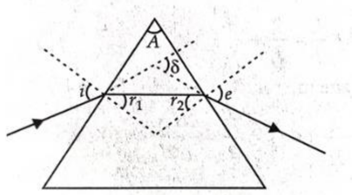
73) c Potential of 20 V will be same across each resistance.
$I_1 = \frac{V}{R_1} = \frac{20}{2} = 10\ A$
$I_2 = \frac{V}{R_2} = \frac{20}{4} = 5\ A$
$I_3 = \frac{V}{R_3} = \frac{20}{5} = 4\ A$
Total current drawn from the circuit,
$I = I_1 + I_2 + I_3 = 10 + 5 + 4 = 19\ A$

74) b The counter torque to prevent the coil from turning will be equal and opposite to the torque acting on the coil.
$\tau = NIAB\sin\theta = NI\pi r^2 B \sin 30^{\circ} = 70 \times 8 \times 3.14 \times (5 \times 10^{-2})^2 \times 1.5 \times \frac{1}{2} = 3.3\ Nm$

75) c The energy stored in the inductor is:

$U = \frac{1}{2}Li^2$
The energy stored in the inductor per second is:
$\frac{dU}{dt} = Li\frac{di}{dt} = 200 \times 10^{-3}\ H \times 1\ A \times 0.5\ A\ s^{-1} = 0.1\ J\ s^{-1}$

76) a Here, $i = 60°, A = 30°, \delta = 30°$
As, $i + e = A + \delta$
$e = A + \delta - i = 30° + 30° - 60° = 0°$
Hence, emergent ray is normal to the surface.
$e = 0° \Rightarrow r_2 = 0°$
As, $r_1 + r_2 = A$
$r_1 = 30° - 0 = 30°$
$\mu = \frac{\sin i}{\sin r_1} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3} = 1.732$



77) b $\beta = \frac{D\lambda}{d} \propto \lambda$
or, $\frac{\beta_2}{\beta_2} = \frac{\lambda_2}{\lambda_1}$
or, $\beta_2 = \frac{\lambda_2}{\lambda_1} \times \beta_1 = \frac{5}{8} \times 2.4 \times 10^{-4} = 1.5 \times 10^{-4}\ m$
Decrease in fringe width $= \beta_1 - \beta_2 = (2.4 - 1.5) \times 10^{-4} = 0.9 \times 10^{-4}\ m$

78) a According to Einstein's photoelectric equation
$K_{max} = \frac{hc}{\lambda} - \emptyset_0$ --- (1)
$2K_{max} = \frac{hc}{\lambda'} - \emptyset_0$ --- (2)
Dividing (2) by (1), we get,
$2 = \frac{\frac{hc}{\lambda'} - \emptyset_0}{\frac{hc}{\lambda} - \emptyset_0}$
$2\frac{hc}{\lambda} - 2\emptyset_0 = \frac{hc}{\lambda'} - \emptyset_0$
$hc\left(\frac{2}{\lambda} - \frac{1}{\lambda'}\right) = \emptyset_0$
$\emptyset_0 = 1240\ \left(\frac{2}{600} - \frac{1}{400}\right) = 1.03\ eV$ [Take hc=1240 eV nm]

79) c $\theta + \beta = \frac{\pi}{2}$
or, $\beta = \frac{\pi}{2} - \theta$
$\cos\theta.\cos\beta = \cos\theta.\cos\left(\frac{\pi}{2} - \theta\right) = \cos\theta.\sin\theta = \frac{1}{2} \times 2\sin\theta\,.\cos\theta = \frac{1}{2}\sin 2\theta$
Since, maximum ($\sin 2\theta = 1$)
maximum value of $\cos\theta.\cos\beta = \frac{1}{2} \times 1 = \frac{1}{2}$

80) a $\cot^{-1}x + \cot^{-1}y = \frac{\pi}{2}$
$\cot^{-1}x = \frac{\pi}{2} - \cot^{-1}y$
$x = \cot\left(\frac{\pi}{2} - \cot^{-1}y\right) = \cot(\tan^{-1}y) = \cot\left(\cot^{-1}\frac{1}{y}\right)$
$x = \frac{1}{y}$
$xy = 1$

81) d $\lim_{x\to0}\frac{1-\cos x}{x^2} = \lim_{x\to0}\frac{2\,sin^2\frac{x}{2}}{x^2} = \lim_{x\to0}\frac{2\left(\sin\frac{x}{2}\right)^2}{\left(\frac{x}{2}\right)^2 \times 2^2} = \frac{2}{4} \times 1 = \frac{1}{2}$

82) a $y = e^{\sqrt{2x}}$
$\frac{dy}{dx} = \frac{d\left(e^{\sqrt{2x}}\right)}{d(\sqrt{2x})}.\frac{d(\sqrt{2x})}{d(2x)}.\frac{d(2x)}{dx} = e^{\sqrt{2x}}.\frac{1}{2}(2x)^{-\frac{1}{2}}.2 = \frac{e^{\sqrt{2x}}}{\sqrt{2x}}$

83) d $\int\frac{dx}{\tan x + \cot x} = \int\frac{dx}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}dx = \int\frac{1}{2}.\frac{\sin 2x}{1}dx = \frac{1}{2}\left[\frac{-\cos 2x}{2}\right] + c = \frac{-\cos 2x}{4} + c$

84) c Given, $\frac{dr}{dt} = 0.25\ cm/sec$
$\frac{dA}{dt} = \frac{d}{dt}(\pi r^2) = \pi \times 2r.\frac{dr}{dt} = \pi \times 2 \times 7 \times 0.25 = 11\ cm^2/sec$

85) a In X-axis, $y = 0$
i.e., $x(1-x)^2 = 0$
$x = 0,1$
Area bounded (A) $= \int_0^1 y\,dx = \int_0^1 x(x^2 - 2x + 1)dx = \left[\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2}\right]_0^1 = \frac{1}{4} - \frac{2}{3} + \frac{1}{2} = \frac{1}{12}$

86) b First series is:

$a + ar + ar^2 + \cdots + \infty$

$\frac{a}{1-r} = 3$ --- (1)

Second series is:

$a^2 + a^2r^2 + a^2r^4 + \cdots + \infty$

$\frac{a^2}{1-r^2} = 3$

$\frac{a.a}{(1-r)(1+r)} = 3$

$\frac{3a}{1+r} = 3$ (from (1))

$\frac{a}{1+r} = 3$ --- (2)

Dividing (1) by (2),

$\frac{1+r}{1-r} = 3$

$r = \frac{1}{2}$

87) $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + xyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (1+xyz)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$

$= 0 \times \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$

88) c $t_{r+1} = {}^6C_r\,(2x)^{6-r}\left(\frac{1}{3x}\right)^r = {}^6C_r\,.\frac{1}{3^r}\,.\,2^{6-r}\,.\,x^{6-2r}$

For term independent of x,

$6 - 2r = 0$

$r = 3$

Required term = ${}^6C_3\,.\frac{2^{6-3}}{3^3} = \frac{160}{27}$

89) a A person can go from A to B and B to C in 5 × 4 = 20 ways

A person can return from C to B and B to A in 3 × 4 = 12 ways (excluding previous ways)

Total number of ways = 20 × 12 = 240

90) a $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, meets y-axis, where $x = 0$,

i.e., $by^2 + 2fy + c = 0$

If y_1 and y_2 are roots of this equation,

$y_1 + y_2 = -\frac{2f}{b}$, $y_1y_2 = \frac{c}{b}$

Length of intercept on y-axis

$= |y_1 - y_2| = \sqrt{(y_1+y_2)^2 - 4y_{1y_2}}$

or, $0 = \sqrt{\frac{4f^2}{b^2} - \frac{4c}{b}}$

or, $0 = \frac{2\sqrt{f^2-bc}}{b}$

$\therefore f^2 = bc$

91) d Given equations of the circles are:

$S_1 : x^2 + y^2 + 4x = 0$

$S_2 : x^2 + y^2 + 2\lambda y = 0$

Equation of the common chord is:

$S_1 - S_2 = 0$

$x^2 + y^2 + 4x - x^2 - y^2 - 2\lambda y = 0$

$2x - \lambda y = 0$ --- (1)

But the equation of the common chord is

$2x - 3y = 0$ --- (2)

(1) and (2) must be identical.

Hence, $\lambda = 3$

92) a distance between the foci, $2ae = 8$

Distance between the directrices, $\frac{2a}{e} = 18$

or, $2ae \times \frac{2a}{e} = 8 \times 18$

$a^2 = 36$

$a = \pm 6$

And $e = \frac{8}{2a} = \frac{8}{2.6} = \frac{2}{3}$

We have : $b^2 = a^2(1 - e^2) = a^2 - a^2e^2 = 36 - 16 = 20$

Required equation is:
$\frac{x^2}{36} + \frac{y^2}{20} = 1$

93) b $QR = \sqrt{(3+1)^2 + (5-3)^2 + (-2-2)^2} = 6$
D.c's of QR are $\frac{3+1}{6}, \frac{5-3}{6}, \frac{-2-2}{6}$ i.e., $\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$

94) a For 4f-orbitals : $n = 4, l = 3, m = -3\ to + 3\ and\ s = \pm 1/2$

95) c $C_2H_5I \xrightarrow{alc.KOH} CH_2 = CH_2 \xrightarrow{Br_2} Br - CH_2 - CH_2 - Br \xrightarrow{KCN} CN - CH_2 - CH_2 - CN$

96) a
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1      2   3  4     5
CH3–C≡C–CH–CH3  (4-Methylpent-2-yne)
            |
           CH3
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97) b $N_1V_1 = N_2V_2$
$V_1 = \frac{N_2V_2}{N_1} = \frac{400 \times 0.1}{0.5} = 80\ ml$
Volume of water added = $V_2 - V_1$ = 400 – 80 =320 ml

98) c $\frac{E_{metal\ chloride}}{E_{metal\ sulphate}} = \frac{W_{metal\ chloride}}{W_{metal\ sulphate}}$
$\frac{x+35.5}{x+48} = \frac{2.67}{3.42}$
$x = \frac{48 \times 2.67 - 35.5 \times 3.42}{3.42 - 2.67} = 9$

99) b In order to complete their octets, the boron trihalides can accept two electrons i.e., one pair of electrons. The order of acidic strength of boron trihalides can be determined on the basis of back-bonding. Due to back-bonding, the electron-density is sent back to the boron atom by the filled orbitals.
Back bonding 2p – 2p 2p – 3p 2p – 4p 2p – 5p
Order of Back bonding:
$BF_3 > BCl_3 > BBr_3 > BI_3$
Stronger is the back bonding weaker is the tendency to act as Lewis acid.
$BF_3 < BCl_3 < BBr_3 < BI_3$

100) c $MnO_4{}^{2-} + e^- \rightarrow MnO_4{}^-$
Oxidation of one mole of $MnO_4{}^{2-}$ requires 1 mole of electrons.
Hence, oxidation of 0.1 mole of $MnO_4{}^{2-}$ will require 0.1 mole of electrons which corresponds to
$0.1 \times 96500 = 9650\ C$

Thank You!!!!!!