IOE MODELENTRANCE EXAM 2023 SET 3

BEATS ENGINEERING

INSTITUTE OF ENGINEERING

Model Entrance Exam

(Set-3 Solutions)

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Section-A (1 marks)

1)	b
2)	a

- 3) d
- 4) c
- 5) a
- 6) c
- 7) a
- 8) a
- 9) a
- 10) d
- 11) a
- 12) c
- 13) a 1 light year = $9.46 \times 10^{15} m$ (Distance travelled by light in 1 year)
- 14) d For a particle performing uniform circular motion, magnitude of the acceleration remains constant.
- 15) a
- 16) b
- 17) c The breaking stress of the wire depends upon the nature of material of the wire.
- 18) a After terminal velocity is reached, the net acceleration of the body falling through a fluid is zero because the body after attaining terminal velocity will continue moving with same velocity through the viscous medium.
- 19) d In a cyclic process, the system returns to its initial state. Since internal energy is a state variable, $\Delta U = 0$, for a cyclic process.
- 20) d No medium is required in radiations, such as radiations from the sun travel through vacuum and reaches us.
- 21) a Speed of sound wave in a fluid is:

$$v = \sqrt{\frac{B}{\rho}}$$

Where, B is the bulk modulus and ρ is the density of the medium.

- 22) a The material suitable for using as a dielectric must have high dielectric strength X and large dielectric constant K.
- 23) b Semiconductors having negative temperature coefficient of resistivity whereas metals are having positive temperature coefficient of resistivity. With increase in temperature, the resistivity of metal increases whereas resistivity of semiconductor decreases.

24) d Under the influence of electric force, the particle moves along electric field. As we know,

Magnetic force $\vec{F} = q(\vec{v} \times \vec{B})$

Here, $\vec{v} \parallel \vec{B}$, so, $\vec{F} = 0$.

Hence, the particle moves along a straight line in the direction of electric field.

25) c As,
$$\varepsilon = L \frac{dI}{dt}$$

When $\frac{dI}{dt} = 1$, $\varepsilon = L$

26) d

27) c When Young's double slit experiment is repeated in water, instead of air

 $\lambda' = \frac{\lambda}{\mu}$, i.e., wavelength decreases.

 $\beta = \frac{\lambda' D}{d}$, i.e., fringe width decreases.

Therefore, fringe becomes narrower.

- 28) b In an unbiased p-n junction, holes diffuse from the p-region to n-region because of the higher hole concentration in p-region than that in n-region.
- 29) c 30) c
- 31) b Sublimation is a process where the compound changes its state from solid to directly into vapours escaping the liquid state. The given mixture is of sand and camphor. Both the components are in solid state. On heating, sand does not change its state but camphor on heating changes to vapor and sand is left behind. Thus, mixture of sand and camphor is purified by sublimation.
- 32) b Fuming sulphuric acid is $H_2S_2O_7$ which is conc. $H_2SO_4 + SO_3$.
- conc. $H_2SO_4 + SO_3 (H_2S_2O_7) \rightarrow$ Fuming sulphuric acid (also known as oleum).
- 33) d $Zn(s) + 2NaOH(aq) \rightarrow 2Na_2ZnO_2(aq) + H_2(g)$ (excess)
- 34) a On moving from left to right in a periodic table, atomic size decreases and on moving from top to bottom, atomic size increases.

35) c Number of moles = $\frac{Weight (gram)}{Molecular weight}$ 36) c $1 \text{ mole} = 6.023 \times 10^{23} \text{ molecules}$ $Fe^{3+} = 1s^2 2s^2 3s^2 3p^6 4s^0 3d^5$ 37) c Here, n = 3, l = 2 (d-subshell) 38) b The conjugate acid of HPO_4^{2-} is $H_2PO_4^{-}$ 39) a $H_2PO_4^- \rightarrow H^+ + HPO_4^{2-}$ The base and its conjugate acid differ by one proton only. $M_2X_3 \rightleftharpoons 2M^{+++} + 3X^{--}$ $K_{sp} (2x)^2 (3x)^3$ 40) d $K_{sn} = (2x)^2 \cdot (3x)^3 = 108 x^5$ $\tan A + \tan B = \frac{a}{b} + \frac{b}{a} = \frac{a^2 + b^2}{ab} = \frac{c^2}{ab}$ 41) d Let $a = 4sin^2\theta + 5cos^2\theta = 4sin^2\theta + 4cos^2\theta + cos^2\theta = 4 + cos^2\theta$ 42) c Least value of $cos^2\theta = 0$ Therefore, Least value of a = 4 + 0 = 4 $\cos^{-1}(2x^2 - 1) + 2\cos^{-1}x = 360^{\circ}$ 43) c $2\cos^{-1}x + 2\cos^{-1}x = 360^{\circ}$ $4\cos^{-1}x = 2\pi$ $\cos^{-1} x = \frac{\pi}{2}$ $x = \cos\frac{\pi}{2} = 0$ $\therefore x \in [-1,0]$ $\vec{a} + \vec{b} + \vec{c} = 0$ 44) a $\vec{b} + \vec{c} = -\vec{a}$ b + c = -a $(\vec{b} + \vec{c})^2 = -(\vec{a})^2$ $b^2 + c^2 + 2bc\cos\theta = a^2$ $\cos\theta = \frac{a^2 - b^2 - c^2}{2bc}$ a + b = p, ab = qNow, $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{p}{q}$ 45) d $x = 1 + \frac{1}{1!} + \frac{4}{2!} + \frac{8}{3!} + \dots = 1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \dots = e^2$ $\therefore x^{-1} = e^{-2}$ 46) c $S_n = \frac{lr - a}{r - 1}$ 255 = $\frac{2(128) - a}{2 - 1}$ 47) a a = 1 $\frac{1+2i}{1-i} = \frac{1+2i}{1-i} \times \frac{1+i}{1-i} = \frac{-1+3i}{2} = \left(-\frac{1}{2}, \frac{3}{2}\right)$ 48) b which lies in II quadrant. $A^{2} + B^{2} = AA + BB = A(BA) + B(AB) = (AB)A + (BA)B = BA + AB = A + B$ 49) c $f[f(\cos 2\theta)] = f\left(\frac{1-\cos 2\theta}{1+\cos 2\theta}\right) = f(tan^2\theta) = \frac{1-tan^2\theta}{1+tan^2\theta} = \cos 2\theta$ 50) c $x^2 + 1 = 0 \Rightarrow x = \pm i \notin R$ 51) b $\Rightarrow B = \phi$ $\lim_{x \to 1} (1-x) \tan \frac{\pi x}{2} = \lim_{x \to 1} \frac{1-x}{\cot \frac{\pi x}{2}} \qquad \left(\frac{0}{0} \ form\right)$ $\lim_{x \to 1} \frac{-1}{-\frac{\pi}{2} \csc^2 \frac{\pi x}{2}} = \frac{1}{\frac{\pi}{2} \times \csc^2 \frac{\pi}{2}} = \frac{1}{\frac{\pi}{2}} = \frac{2}{\pi}$ 52) b $y = x + e^x$ 53) a $\frac{dy}{dx} = 1 + e^x$ $\frac{dx}{d^2y}$ $\frac{dx^2}{dx^2}$ $= e^{x}$ $\frac{\frac{d^2y}{dx^2}}{dx^2} = y - x$ $\therefore y = x + e^x$ $f(x) = x^3 - 3x^2 + 6x + 7$ 54) d $f'(x) = 3x^2 - 6x + 6$

f'(x) = 0 $3x^2 - 6x + 6 = 0$
 $x^2 - 2x + 2 = 0$ $b^2 - 4ac = 4 - 4 \times 1 \times 2 = -4 < 0$ This has no real roots. So, there exist no extreme points. $\int_0^{\pi} \cos^3 x \, dx$ 55) b $= \int_{0}^{\pi} \frac{\cos 3x + 3\cos x}{4} \qquad \because \cos 3x =$ = $\frac{1}{4} \left| \frac{\sin 3x}{3} + 3\sin x \right|_{0}^{\pi} = \frac{1}{4} [0 + 0] = 0$ $:: \cos 3x = 4\cos^3 x - 3\cos x$ Any line perpendicular to 3x - 2y = 6 is 56) c $\frac{2x + 3y = k}{\frac{x}{(k/2)} + \frac{y}{(k/3)}} = 1$ Given, y-intercept = 2 $\frac{k}{3} = 2$ k = 6 $\therefore \text{ x-intercept} = \frac{k}{2} = \frac{6}{2} = 3$ Applying condition of orthogonality, 57) a $2g_1g_2 + 2f_1f_2 = c_1 + c_2$ $2\left(\frac{a}{2}\right)\left(-\frac{3}{2}\right) + 2(0)\left(\frac{1}{2}\right) = 1 + 5$ -3a = 12a = -4Given, ae = 1, a = 258) c 2e = 1 $e = \frac{1}{2}$ Also, $ae = \sqrt{a^2 - b^2}$ $(ae)^2 = a^2 - b^2$ $1 = 4 - b^2$ $b^{2} = 3$ $b = \sqrt{3}$ \therefore Minor axis = $2b = 2\sqrt{3}$ Product of slopes of given asymptotes $=\frac{-3}{4} \times \frac{4}{3} = -1$ 59) c

Since the asymptotes are perpendicular to each other, the hyperbola is a rectangular hyperbola, whose eccentricity = $\sqrt{2}$ 60) c Dr's of line joining given points are:

4-(-2), 3-1, -5-(-8) i.e., 6, 2, 3

$$\therefore$$
 Dc's are:
 $\frac{6}{\sqrt{6^2+2^2+3^2}}, \frac{2}{\sqrt{6^2+2^2+3^2}}, \frac{3}{\sqrt{6^2+2^2+3^2}}$ i.e., $\frac{6}{7}, \frac{2}{7}, \frac{3}{7}$

Section-B (2 marks)

61) a

- 62) d
- 63) c

64) b

65) d Velocity of train, v_T = 72 × ¹⁰⁰⁰/₃₆₀₀ = 20 m/s Velocity of bird, v_B = 18 × ¹⁰⁰⁰/₃₆₀₀ = 5 m/s Since, bird is flying parallel to train in opposite direction, relative velocity of bird w.r.t. train = v_B + v_T = 25 m/s Train's length = 175 m Time taken by the bird to cross the train = ¹⁷⁵/₂₅ = 7 s
66) a Moment of inertia of the solid cylinder about its axis is L = ^{MR²}/₂ = ^{(20)(20×10⁻²)²} = 0.4 kg m²

$$I = \frac{MR^2}{2} = \frac{(20)(20 \times 10^{-2})}{2} = 0.4 \ kg \ m^2$$

Angular momentum $L = I\omega = (0.4) \times (100) = 40 J s$ 67) d According to Archimedes' principle,

Weight of the body = Weight of the liquid displaced Let V be the volume of block.

In water, $V \rho_{block} g = \left(\frac{4}{5}V\right) \rho_{water} g$ $\rho_{block} = \frac{4}{5} \rho_{water}$ --- (1) In liquid, $V \rho_{block} g = V \rho_{liquid} g$ $\rho_{block} = \rho_{liquid}$ From (1) and (2), --- (2) $\rho_{liquid} = \frac{4}{5}\rho_{water} = \frac{4}{5} \times 10^3 = 800 \ kg \ m^{-3}$ $v = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ 68) c $v^2 = \left(\frac{1}{2\pi}\sqrt{\frac{k}{m}}\right)^2$ $k = 4\pi^2 v^2 m = 4\pi^2 \left(\frac{5}{\pi}\right)^2 \times 20 \times 10^{-3} = 2 Nm^{-1}$ If the car approaches the policeman $v' = \frac{v+v_0}{v-v_s} \times v = \frac{300+15}{300-0} \times 500 = 525 Hz$ 69) b If the car moves away from the policeman $v'' = \frac{v - v_0}{v - v_s} \times v = \frac{300 - 15}{300 - 0} \times 500 = 475 Hz$ Change in frequency = v' - v'' = 525 - 475 = 50 Hz70) d Mass of water, $m_w = 100 \text{ g}$ Mass of ice, $m_i = 10$ g Specific heat of water, $s_w = 1 \ cal \ g^{-1} \ c^{-1}$ Latent heat of fusion of ice, $L_{fi} = 80 \ calg^{-1}$ Let T be the final temperature of the mixture. According to principle of calorimetry, Heat lost by water = Heat gained by ice $m_w s_w (\Delta T)_w = m_i L_{fi} + m_i s_w (\Delta T)_i$ $100 \times 1 \times (50 - T) = 10 \times 80 + 10 \times 1 \times (T - 0)$ 500 - 10T = 80 + T $T = \frac{420}{11} = 38.2 \ ^{\circ}C$ 71) d Since the gas is suddenly expanded, it means the process is adiabatic process. $T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$ $(273 + 20) \times V_1^{\gamma - 1} = T_2 \times (8V_1)^{\gamma - 1}$ $293 = T_2 \times 8^{\gamma - 1}$ $T_2 = \frac{293}{\frac{5}{8^3 - 1}} = \frac{293}{\frac{2}{8^3}} = \frac{293}{(2^3)^{\frac{2}{3}}} = \frac{293}{4} = 73.25 \text{ K}$ 72) c Here, distance of point from the centre of sphere, r = 20 cm = 0.2 mElectric field, $E = -1.2 \times 10^3 NC^{-1}$ As, $E = \frac{q}{4\pi\varepsilon_0 r^2}$ $q = (4\pi\varepsilon_0 r^2)E = \frac{(0.2)^2 \times (-1.2 \times 10^3)}{9 \times 10^9} = -5.3 \times 10^{-9} C$ Here, in the balanced condition of potentiometer, 73) a $\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$ $\varepsilon_2 = \varepsilon_1 \times \frac{l_2}{l_1} = 1.5 \times \frac{65}{32} = 3.05 V$ Torque $(\tau)^{\iota_1} = MB \sin \theta$ $M = \frac{\tau}{B \sin \theta} = \frac{4.5 \times 10^{-2}}{0.35 \times \sin 30^\circ} = \frac{4.5 \times 10^{-2}}{0.35 \times \frac{1}{2}} = 0.26 JT^{-1}$ 74) c 75) d $R = 0.2 \ k\Omega = 200 \ \Omega$ Capacitive reactance, $X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.14 \times 50 \times 15 \times 10^{-6}} = 212 \ \Omega$ The impedance of the RC circuit is: $Z = \sqrt{R^2 + X_c^2} = \sqrt{(200)^2 + (212)^2} = 291.5 \,\Omega$ $\mu_{1} = 1, R = 20 \text{ cm}, u = -100 \text{ cm}, \mu_{2} = 1.5$ $\frac{\mu_{2}}{v} - \frac{\mu_{1}}{u} = \frac{\mu_{2} - \mu_{1}}{R}$ $\frac{1.5}{v} + \frac{1}{100} = \frac{1.5 - 1}{20}$ $\frac{1.5}{v} = \frac{1}{40} - \frac{1}{100}$ 76) c

$$\frac{1}{2} = \frac{1}{2} = \frac{1}$$

89) c 90A + 5H = 918--- (1) We have, $AM \times HM = GM^2$ AH = 36--- (2) Solving (1) and (2), we get, $90A + 5\left(\frac{36}{A}\right) = 918$ $90A^2 - 918A + 180 = 0$ $5A^2 - 51A + 10 = 0$ $5A^2 - 50A - A + 10 = 0$ $A = 10, \frac{1}{5}$ $\frac{1+i}{1-i} = \frac{\binom{5}{2}+i}{1-i} = \frac{i(1-i)}{1-i} = i$ 90) b $\frac{1-i}{1+i} = \frac{1-i}{1+i} = \frac{1-i}{1+i} = \frac{1-i}{1+i} = -1$ $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy$ $i^3 - (-i)^3 = -i - i = -2i = 0 - 2i$ (x, y) = (0, -2)91) b Two particular persons can sit together in 2! ways. After their sitting, remaining 18 persons can occupy their seats in 18! ways. So, required number = $18! \times 2!$ $\lim_{x \to \infty} kx \ cosec \ x = \lim_{x \to \infty} kx \ coseckx$ 92) c $\lim_{x \to 0} k\left(\frac{x}{\sin x}\right) = \lim_{x \to 0} \left(\frac{kx}{\sin kx}\right) \cdot \frac{1}{k}$ $k.1 = 1.\frac{1}{k}$ $k^{2} = 1$ $k = \pm 1$ 93) b $y = \sqrt{x \log_e x}$ At $x = e, y = \sqrt{e}$ Now, $y^2 = x \log_e x$ Differentiating both sides w.r.t. x, we get, $2y \cdot \frac{dy}{dx} = 1 + \log_e x$ $\frac{dy}{dx} = \frac{1}{2y}(1 + \log_e x)$ At x = e, $\frac{dy}{dx} = \frac{1}{2\sqrt{e}} \cdot (1+1) = \frac{1}{\sqrt{e}}$ $f(x) = \sin x (1 + \cos x) = \sin x + \frac{1}{2} \sin 2x$ 94) b $f'(x) = \cos x + \cos 2x$ $f''(x) = -\sin x - 2\sin 2x$ Now, f'(x) = 0 $\cos x + \cos 2x = 0$ $\cos 2x = -\cos x = \cos(\pi - x)$ $2x = \pi - x$ $x = \frac{\pi}{3}$ Also, $f''\left(\frac{\pi}{3}\right) = \frac{-\sqrt{3}}{2} - \sqrt{3} < 0$ \therefore f(x) has a maxima at $x = \frac{\pi}{3}$ $\int \frac{dx}{e^{x} + e^{-x} + 2} = \int \frac{e^{x} dx}{e^{2x} + 1 + 2e^{x}} = \int \frac{e^{x} dx}{(e^{x})^{2} + 2e^{x} \cdot 1 + (1)^{2}} = \int \frac{e^{x} dx}{(e^{x} + 1)^{2}} = \int (e^{x} + 1)^{-2} \cdot e^{x} dx$ $= \frac{(e^{x} + 1)^{-2 + 1}}{e^{-2 + 1}} + c \quad \because \int [f(x)]^{n} \cdot f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + c$ 95) b $=\frac{1}{\frac{-1}{e^{x}+1}+c}$ 96) d (0,1) (-1,0 (1.0)

(0,-1)

|x| + |y| = 1 represent a square with length of each diagonal = 2 $\therefore A = \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 2 \times 2 = 2$ The line through (2,3) is 3x - 2y = 0 and through (4,5) is 5x - 4y = 0. 97) b Joint equation is: (3x - 2y)(5x - 4y) = 0 $15x^2 - 22xy + 8y = 0$ Comparing with $ax^2 + 2hxy + by^2 = 0$ a = 15, 2h = -22, b = 8 $\therefore a + 2h + b = 15 - 22 + 8 = 1$ 98) c Given circle is: $2x^{2} + 2y^{2} - 3x + 6y + 2 = 0$ $x^{2} + y^{2} - \frac{3}{2}x + 3y + 1 = 0$ $2g = -\frac{3}{2} \Rightarrow g = -\frac{3}{4}$ $2f = 3 \Rightarrow f = \frac{3}{2}$ radius (r) = $\sqrt{g^2 + f^2 - c} = \sqrt{\frac{9}{16} + \frac{9}{4} - 1} = \frac{\sqrt{29}}{4}$ Given, $A_2 = 2A_1$ $\pi R^2 = 2 \times \pi \times \frac{29}{16}$ $R^2 = \frac{29}{8}$ Therefore, equation of required circle is: $\left(x - \frac{3}{4}\right)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{29}{8}$ $\frac{16x^2 - 24x + 9}{16} + \frac{4y^2 + 12y + 9}{4} - \frac{29}{8} = 0$ $\frac{16x^2 - 24x + 9 + 16y^2 + 48y + 36 - 58}{16} = 0$ $16x^2 + 16y^2 - 24x + 48y - 13 = 0$ $y^2 - y + x = 0 \quad \dots \quad (1)$ 99) c Differentiating both sides w.r.t. x, we get, $2y \cdot \frac{dy}{dx} - \frac{dy}{dx} + 1 = 0$ $\frac{dy}{dx} = \frac{1}{1 - 2y}$ But slope of tangent x + y - a = 0 is -1. $\therefore -1 = \frac{1}{1-2y}$ y = 1From (1), 1 - 1 + x = 0x = 0Hence, point of contact is (0, 1). 100) a As given, $\frac{2-6-1+k}{\sqrt{14}} \cdot \frac{1+4-3}{\sqrt{14}} = 1$ k - 5 = 7k = 12

Thank You!!!!!!