INSTITUTE OF ENGINEERING

Model Entrance Exam

(Set-6 Solutions)

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Date: 2080/03/23
(July-08)

Duration: 2 hours
Time: 8 AM – 10 AM

Section-A (1 marks)

1) b
2) d
3) d
4) c
5) d
6) b
7) b
8) d
9) a
10) d
11) a
12) a
13) a Electric potential, $V = \frac{W}{q} = \frac{[ML^2T^{-2}]}{[AT]} = [ML^2T^{-3}A^{-1}]$
14) c Order of velocity : $\gamma > \beta > \alpha$
Particle with high velocity stay for very short time near the atom of medium so possibility of ionization is less energy loss is gradual so they can penetrate more.
$\therefore Penetrating\ power :\ \gamma > \beta > \alpha$
15) b
16) c Lenz's law gives direction of induced current.
17) c
18) d As the electric field is uniform, equal force acts throughout the field i.e., the lines of force are parallel.
19) c Air column vibrates, not the liquid column.
20) b
21) d
22) a $\left(\frac{dQ}{dt}\right) = \sigma AT^4 \propto A$
Sphere has minimum surface area, so rate of cooling is slowest.
23) c
24) a Spring balance now reads the wight of both stone and container.
25) b
26) b Acceleration of spaceship towards earth's center:
$a_c = \frac{{v_0}^2}{r} = \frac{GM}{r^2}$
$\therefore R = m(g' - a) = \left(\frac{GM}{r^2} - \frac{GM}{r^2}\right) = 0$ (weightlessness)
27) b In parallel combination of cells, the voltage across the terminals is same and resistance is minimum. Therefore, from $V = IR$, the current drawn from cell combination will be more.
28) a $\Delta p = mv - (-mv) = 2mv$
29) b
30) a $\lim_{x\to 0} \frac{e^{\sin x}-1}{x} = \lim_{x\to 0} \frac{e^{\sin x}-1}{\sin x} \times \frac{\sin x}{x} = 1 \times 1 = 1$
31) c $\frac{d}{dx}[\log(\sin x)] = \frac{d[\log(\sin x)]}{d(\sin x)} \times \frac{d(\sin x)}{x} = \frac{1}{\sin x} \times \cos x = \cot x$
32) c $f(x) = \begin{cases} x - 1, & x \text{ is even} \\ x + 1, & x \text{ is odd} \end{cases}$
which is clearly one-one and onto.
33) d $y^2 = 4x \Rightarrow 2y\frac{dy}{dx} = 4$
$\frac{dy}{dx} = \frac{2}{y}$
At (1, 2),
$\frac{dy}{dx} = \frac{2}{2} = 1$
$\therefore$ Equation of normal at (1, 2) is:
$y - 2 = -1(x - 1)$
$x + y - 3 = 0$
34) c $\int(x\cot x + \log\sin x)dx = x\log(\sin x) + c$ $(\because \int[x\,f'(x) + f(x)]dx = x\,f(x) + c)$
35) c Here, $\alpha + \beta = p, \alpha\beta = 8$
Given, $|\alpha - \beta| = 2$
$(\alpha - \beta)^2 = 4$

$(\alpha+\beta)^2-4\,\alpha\beta=4$
$p^2-4\times 8=4$
$p^2=36$
$p=\pm 6$

36) b $T_n=S_n-S_{n-1}$
$164=(3n^2+5n)-[3(n-1)^2+5(n-1)]$
$164=6n+2$
$n=27$

37) c Given expression $=(1-i)^n\left(1+\frac{i^2}{i}\right)^n=(1-i)^n(1+i)^n=(1-i^2)^n=(1-(-1))^n=2^n$

38) a Required number of diagonals = $^nC_2-n$
When, $n=7$
$^7C_2-7=\frac{7\times 6}{2}-7=21-7=14$

39) d $A=\{2,4,6,8,\dots\}, B=\{1,3,5,7,\dots\}$
$A\cap B=\phi$
$(A\cap B)'=U$

40) a $P^2+2P-3I=\begin{bmatrix}2&-1\\1&2\end{bmatrix}\begin{bmatrix}2&-1\\1&2\end{bmatrix}+2\begin{bmatrix}2&-1\\1&2\end{bmatrix}-3\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}4-1&-2-2\\2+2&-1+4\end{bmatrix}+\begin{bmatrix}4&-2\\2&4\end{bmatrix}-\begin{bmatrix}3&0\\0&3\end{bmatrix}$
$=\begin{bmatrix}3+4-3&-4-2-0\\4+2-0&3+4-3\end{bmatrix}=\begin{bmatrix}4&-6\\6&4\end{bmatrix}$

41) c Given equation can be written as:
$x^2+2(2)xy+ky^2=0$
It represents coincident lines if $h^2-ab=0$
$2^2-1\times k=0$
$k=4$

42) b The general equation $x^2+y^2+2gx+2fy+c=0$ represents:
1) a real circle if $g^2+f^2>c$
2) a point circle if $g^2+f^2=c$
3) an imaginary circle if $g^2+f^2<c$

43) a Given line is:
$3x-4y+5=0$
$y=\frac{3}{4}x+\frac{5}{4}$
The line is tangent to the parabola iff $c=\frac{a}{m}$
i.e., $\frac{5}{4}=\frac{a}{\frac{3}{4}}$
$a=\frac{5}{4}\times\frac{3}{4}=\frac{15}{16}$

44) d $\frac{(x-3)^2}{9}+\frac{(y-5)^2}{25}=1$
$\frac{(x-3)^2}{3^2}+\frac{(y-5)^2}{5^2}=1$
Here, $a=3, b=5$
Here, $b>a\Rightarrow$ major axis is parallel to y-axis.
Length of major axis $=2b=2\times 5=10$

45) b Required line is y-axis whose dc's are 0, 1, 0.

46) d Given expression $=4\sin A\cos^3 A-4\cos A\sin^3 A$
$=4\cos A\sin A\,(cos^2A-sin^2A)=2(2\sin A\cos A)(\cos 2A)=2(\sin 2A)(\cos 2A)=\sin 4A$

47) b $|\sin x|=\cos x$ --- (1)
$sin^2x=cos^2x$
$2cos^2x=1$
$\cos x=+\frac{1}{\sqrt{2}}$ [$\because \cos x$ cannot be negative by (1)]
$x=2n\pi\pm\frac{\pi}{4}$

48) c $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$
$\sin^{-1}\frac{1}{5}+\cos^{-1}x=\frac{\pi}{2}$
$\cos^{-1}x=\frac{\pi}{2}-\sin^{-1}\frac{1}{5}$
$\cos^{-1}x=\cos^{-1}\frac{1}{5}$
$x=\frac{1}{5}$

49) c $\left|\vec{a}+\vec{b}\right|^2=a^2+b^2+2.\vec{a}.\vec{b}=2(1+\cos\theta)=2.2\cos^2\frac{\theta}{2}$

$|\vec{a}+\vec{b}| = 2\cos\frac{\theta}{2}$

50) a The boiling points of benzene(351K) and chlorobenzene(405K) differ by 54K, i.e., more than 30K and hence these can be separated by simple distillation.

51) d Only RCH_2NO_2 has α-hydrogen atoms and hence shows tautomerism.

52) a Temporary hardness is due to the presence of soluble bicarbonates of calcium and magnesium in water which can be removed by boiling the water.
Permanent hardness is due to the presence of sulphates and chlorides of calcium and magnesium.

53) b $H_2SO_4 \xrightarrow{\Delta} H_2O + SO_3$

54) a

55) b

56) c

57) a

58) a

59) d

60) a Greater the electronegativity difference, more polar is the bond.

<u>Section-B</u> (2 marks)

61) b The passage states that Portuguese explorers thought the coconut resembled a face, because of its three dimples (like eyes and mouth) and its hairy texture.

62) b The passage does touch briefly on most of these choices, but the main focus is on the many ways that coconut palms are used.

63) c All of the choices are included in the passage except helium balloons, which are not mentioned.

64) d The word staples in this context refers to commonly used items. The best choice for "diet staples" is foods.

65) a Relative velocity of B w.r.t. A $= v_A + v_B = 50\ m/s$ (trains move in opposite directions)
Total path length to be covered by B $= 130 + 120 = 250\ m$
$\therefore$ Time taken by train B $= \frac{250}{50} = 5\ s$

66) a Retardation while sliding on a rough horizontal surface, is μg. So,
$0^2 = u^2 - 2as$
$0 = 10^2 - 2\ \mu g \times 50$
$\mu = \frac{100}{1000} = 0.1$

67) c From conservation of momentum,
$12 \times 0 = 4v_1 + 8 \times 6$
$\therefore v_1 = -12\ m/s$
The minus sign indicates opposite direction.
$K.E. = \frac{1}{2} \times 4 \times 12^2 = 288$ J

68) c $\frac{T'}{T} = \sqrt{\frac{g}{1.02g}} = 0.99$
$T' = 0.99\,T$

69) b $\frac{v_1}{v_2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{2}{3}\right)^2 = 4:9$

70) c $mc(t-\theta) = \frac{m}{2} \times 2c(2\theta - t)$
$\therefore t = (3/2)\theta$

71) a $E \propto T$
$E_2 = \frac{T_2}{T_1} \times E_1 = \frac{327+273}{27+273} E_1 = 2E_1$

72) d $f = \frac{D^2-d^2}{4D} = \frac{50^2-10^2}{4\times 50} = 12$ cm

73) d The defect is myopia which is removed by using convex lens of focal length f.
$f = \frac{xd}{x-d} = \frac{40\times 25}{40-25} = \frac{200}{3} cm = \frac{2}{3} m$
$\therefore P = \frac{1}{f} = \frac{3}{2} = 1.5$ D

74) a Maximum particle velocity = $\omega A = 2\pi f A = 2\pi \times 300 \times 0.1 = 60\pi\ cm/s$

75) d $W = Fs\,cos\theta = qEs\,cos\theta$
$\therefore E = \frac{W}{qs\,cos\theta} = \frac{4}{0.2\times 2\times cos60^\circ} = 20$ N/C

76) c Current by the cell, $i = \frac{E}{R+R'} = \frac{3}{20+10} = \frac{1}{10}$ A

P.d. across the wire $V = 20 \times \frac{1}{10} = 2$ V

Potential gradient $\frac{dV}{dr} = \frac{2}{10} = 0.2$ Vm^{-1}

77) d $W = \int_0^\theta \tau d\theta = \int_0^\theta MB sin\theta\, d\theta$

$= MB(cos 0 - cos\theta)$

$= MB(1 - cos\theta)$

78) a $^1H_2 + {}^1H_2 \rightarrow {}^2He_4 + \Delta E$

The binding energy per nucleon of a deuterium = 1.1 MeV

∴ Total binding energy (E_1) = $2 \times 1.1 = 2.2$ MeV

The binding energy per nucleon of a helium nuclei = 7.0 MeV

∴ Total binding energy (E_1) = $4 \times 7 = 28$ MeV

Energy released (ΔE) $= E_2 - 2E_1 = 28 - 2 \times 2.2 = 23.6$ MeV

79) a Let $\sin^{-1} x = \theta$. Then, $x = \sin\theta$.

Now, $x \rightarrow \frac{1}{\sqrt{2}} \Rightarrow \sin\theta \rightarrow \frac{1}{\sqrt{2}} \Rightarrow \theta \rightarrow \frac{\pi}{4}$.

$\lim_{x\to\frac{\pi}{2}} \frac{x - \cos(\sin^{-1}x)}{1 - \tan(\sin^{-1}x)} = \lim_{\theta\to\frac{\pi}{4}} \frac{\sin\theta - \cos\theta}{1 - \tan\theta} = \lim_{\theta\to\frac{\pi}{4}} \frac{\sin\theta - \cos\theta}{\cos\theta - \sin\theta} \times \cos\theta = \lim_{\theta\to\frac{\pi}{4}} -\cos\theta = -\frac{1}{\sqrt{2}}$

80) d $y = a\sin x + b\cos x$

$\frac{dy}{dx} = a\cos x - b\sin x$

Now, $\left(\frac{dy}{dx}\right)^2 = (a\cos x - b\sin x)^2 = a^2cos^2x + b^2sin^2x - 2ab\sin x.\cos x$

And, $y^2 = (a\sin x + b\cos x)^2 = a^2sin^2x + b^2cos^2x + 2ab\sin x.\cos x$

So, $y^2 + \left(\frac{dy}{dx}\right)^2 = a^2cos^2x + b^2sin^2x - 2ab\sin x.\cos x + a^2sin^2x + b^2cos^2x + 2ab\sin x.\cos x$

$= a^2(sin^2x + cos^2x) + b^2(sin^2x + cos^2x) = a^2 + b^2 = constant$

81) a $\cos 60° = \frac{a^2+b^2-c^2}{2ab}$

$\frac{1}{2} = \frac{a^2+b^2-c^2}{2ab}$

$a^2 + b^2 - c^2 = ab$

$a^2 + b^2 = ab + c^2$

Now, $\frac{a}{b+c} + \frac{b}{c+a} = \frac{ac+a^2+b^2+bc}{bc+ab+c^2+ac} = \frac{ac+ab+c^2+bc}{ac+ab+c^2+bc} = 1$

82) d $2sin^2\theta - 3\sin\theta - 2 = 0$

$(\sin\theta - 2)(2\sin\theta + 1) = 0$

$\sin\theta = -\frac{1}{2}$ [∵ $\sin\theta = 2$ is not possible]

$\sin\theta = \sin\frac{7\pi}{6}$

$\theta = n\pi + (-1)^n\frac{7\pi}{6}$

83) a Let V be the volume of cylinder at any time t.

Then, $V = \pi r^2 h$

$\frac{dV}{dt} = \pi\left\{2rh.\frac{dr}{dt} + r^2\frac{dh}{dt}\right\}$

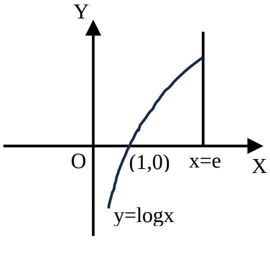
$0 = \pi\left(-2rh.\frac{dh}{dt} + r^2\frac{dh}{dt}\right)$ $\left[∵ \text{V = constant and} \frac{dr}{dt} = -\frac{dh}{dt}\right]$

$r^2\frac{dh}{dt} = 2rh.\frac{dh}{dt}$

$r = 2h$

84) a $\int \frac{\sqrt{\tan x}}{2\sin x.\cos x}dx = \int \frac{\sqrt{\tan x}}{2\sin x.\cos x} \times \frac{sec^2x}{sec^2x}dx = \int \frac{\sqrt{\tan x}.sec^2x}{2\tan x}dx = \frac{1}{2}\int\frac{sec^2x}{\sqrt{\tan x}}dx = \frac{1}{2} \times 2\sqrt{\tan x}$

$= \sqrt{\tan x} + c$ $\left[∵ \int\frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + c\right]$



85) b $Area = \int_1^e ydx = \int_1^4 \log x\, dx = [x\log x - x]_1^e = (e\log e - e) - (1\log 1 - 1) = 1$

86) d $T_{13} = {}^nC_{12}\,(x^2)^{n-12}\left(\frac{2}{x}\right)^{12} = {}^nC_{12}\,x^{2n-24-12}.2^{12} = {}^nC_{12}\,x^{2n-36}.2^{12}$

For term independent of x,

$2n - 36 = 0$

$n = 18$

The divisors of $n = 18$ are 1, 2, 3, 6, 9, 18.

Sum = $1 + 2 + 3 + 6 + 9 + 18 = 39$

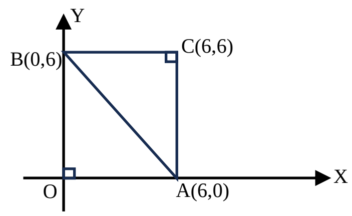
87) b $\log 2 - \frac{\log 4}{2^2} + \frac{\log 8}{3^2} - \frac{\log 16}{4^2} + \cdots$

$= \log 2 - \frac{2\log 2}{2^2} + \frac{3\log 2}{3^2} - \frac{4\log 2}{4^2} + \cdots$

$= \log 2\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\right)$
$= \log 2 \times \log 2$
$= (\log 2)^2$

88) d $\tan\theta = \frac{1-\cos\frac{\pi}{5}}{\sin\frac{\pi}{5}} = \frac{2sin^2\left(\frac{\pi}{10}\right)}{2\sin\frac{\pi}{10}.\cos\frac{\pi}{10}} = \tan\frac{\pi}{10}$
$\theta = \frac{\pi}{10}$

89) d f(x) is defined when $\log\left(\frac{5x-x^2}{4}\right) \geq 0$
$\left(\frac{5x-x^2}{4}\right) \geq e^0$
$5x - x^2 \geq 4$
$x^2 - 5x + 4 \leq 0$
$1 \leq x \leq 4$
$x \in [1,4]$

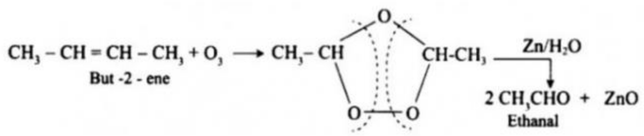


90) c If given vertices are A, B, C respectively, then we find that ΔABC is right angled at C.
Circumcenter = Mid-point of AB = $\left(\frac{6+0}{2}, \frac{0+6}{2}\right) = (3,3)$
Centroid = $\left(\frac{6+0+6}{3}, \frac{0+6+6}{3}\right) = (4,4)$
Required distance = $\sqrt{(4-3)^2 + (4-3)^2} = \sqrt{2}$

91) d Equation of circle is given by:
$(x+4)(x-12) + (y-3)(y+1) = 0$
$x^2 + y^2 - 8x - 2y - 51 = 0$
Length of y-intercept $= 2\sqrt{f^2 - c} = 2\sqrt{1+51} = 2\sqrt{52} = 4\sqrt{13}$

92) c For ellipse, $e_e = \sqrt{1 - \frac{b^2}{16}}$
For hyperbola, $e_h = \sqrt{1 + \frac{225}{400}} = \frac{5}{4}$
Since, their directrices are same,
$\left(\frac{a}{e}\right)_{ellipse} = \left(\frac{a}{e}\right)_{hyperbola}$
$\frac{4}{e_e} = \frac{10}{\frac{5}{4}}$
$\frac{4}{e_e} = 8$
$e_e = \frac{1}{2}$
${e_e}^2 = \frac{1}{4}$
$1 - \frac{b^2}{16} = \frac{1}{4}$
$\frac{b^2}{16} = \frac{3}{4}$
$b^2 = 12$

93) c Equation of plane is:
$\frac{x}{2} - \frac{y}{8} + \frac{z}{6} = 1$
$12x - 3y + 4z = 24$
It's distance from (2, -3, 1) $= \frac{24+9+4-24}{\sqrt{144+9+16}} = \frac{13}{13} = 1$

94) a


1 2 3 4 5
95) b $H_3C - CH = C = CH - H_3C$
C_2 : sp^2-hybridized, C_3 : sp-hybridized

96) a $M_1V_1 + M_2V_2 = M_3V_3$
$0.5 \times 750 + 2 \times 250 = M_3 \times (750 + 250)$
$M_3 = \frac{375+500}{1000} = 0.875\ M$

97) a $2\ MnO_4^{\ -} + 5\ H_2O_2 \rightarrow 2\ Mn^{2+} + 5H_2O + 9O_2 + 6\ e^-$

98) b Electricity passed $= 3.86 \times (41 \times 60 + 40) = 3.86 \times 2500 = 9650$
$Ca^{2+} + 2e^- \rightarrow Ca$

2×96500 C of electricity deposit calcium $= 40\ g$
9650 C of electricity will deposit calcium $= \frac{40}{2\times96500} \times 9650 = 2\ g$

99) c For isoelectronic species, ionic radii decrease with increase in effective (relative) charge. Also, Ar, K and Ca belong to the same period (3rd period).
Thus, the order is: $Ca^{2+} < K^{+} < Ar$

100) a **Key Concept** : When equal volumes of acid and base are mixed, then resulting solution becomes alkaline if concentration of base is taken high.
Let normality of solution after mixing 0.1 M NaOH and 0.01 M HCl is N.
$N_1V_1 - N_2V_2 = NV$
or, $0.1 \times 1 - 0.01 \times 1 = N \times 2$
Since, normality of NaOH is more than that of HCl, the resulting solution is alkaline.
$[OH]^- = N = \frac{0.09}{2} = 0.45$ N
$pOH = -\log(0.45) = 1.35$
$\therefore pH = 12 - 1.35 = 12.65$

Thank You!!!!!!