BEATS ENGINEERING

INSTITUTE OF ENGINEERING

Model Entrance Exam

(Set-7 Solutions)

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Date: 2080/03/30
(July-15)

Duration: 2 hours
Time: 8 AM – 10 AM

<u>Section-A</u> (1 marks)

1) c
2) d
3) b
4) b
5) d
6) a
7) c
8) b
9) b
10) b
11) b
12) b
13) c $[X] = [Force] \times [Density] = [MLT^{-2}] \times [ML^{-3}] = [M^2L^{-2}T^{-2}]$
14) d The area under the velocity-time graph represents the displacement over a given time interval.
15) b Action and reaction force always act in pairs and on different bodies.
16) d When the total external force acting on the system is zero, the velocity of centre of mass remains constant.
17) d Linear momentum is not conserved.
18) d The circular motion of a particle with constant speed is periodic but not simple harmonic motion as the particle repeats its motion after a regular interval of time but does not oscillate about a fixed point.
19) c In a given P-V diagram, pressure remains constant although volume increases. Hence, the process is an isobaric process.
20) b According to Wein's law,
$\lambda_m T = \text{constant}$
$\lambda_m \propto T^{-1}$
21) d The phenomenon of beats can take place for both longitudinal and transverse waves.
22) d Electric field at a point is continuous if there is no charge at that point, and the field is discontinuous if there is charge at that point.
23) d The force between two parallel current carrying wires is independent of the radii of the wires.
24) c Core of electromagnets are made of soft iron that is a ferromagnetic material with high permeability and low retentivity.
25) b
26) c $\beta = \frac{\lambda D}{d}$
If we replace yellow light with blue light, i.e., longer wavelength with shorter one, the fringe width decreases.
27) d Resolving power $\propto$ aperture
28) b $\lambda = \frac{h}{p} = \frac{h}{mv}$
If the velocity of the electron increases, de Broglie wavelength decreases.
29) d Electrons are the majority charge carriers in n-type semiconductors.
30) b As given, sum of roots $= -3$
$-\left(\frac{2a+3}{a+1}\right) = -3$
$2a + 3 = 3a + 3$
$a = 0$
Product of roots $= \frac{3a+4}{a+1} = \frac{3(0)+4}{0+1} = 4$
31) c $1 + \frac{(\log x)^2}{2!} + \frac{(\log x)^2}{4!} + \cdots = \frac{e^{\log x} + e^{-\log x}}{2} = \frac{x + x^{-1}}{2}$
32) a $S_n = \frac{lr - a}{r - 1}$
$255 = \frac{2(128) - a}{2 - 1}$
$a = 1$
33) a $(1 - i)x + (1 + i)y = 1 - 3i$
$(x + y) + (-x + y)i = 1 - 3i$
Equating real and imaginary parts, we get

$x + y = 1, -x + y = -3$
On solving, we get,
$x = 2, y = -1$

34) c A square matrix A of order $n \times n$ is called a/an:
- Singular matrix if $|A| = 0$
- Non-singular matrix if $|A| \neq 0$
- Symmetric matrix if $A^T = A$
- Skew symmetric matrix if $A^T = -A$

35) b There are two alternatives for the button of each bulb 'on' and 'off'. To enlight the hall at least one bulb button should be 'on'.
Total no. of ways $= 2^{10} - 1 = 1023$

36) d The period of $\cos 4x$ is $\frac{\pi}{2}$ and that of $\tan 3x$ is $\frac{\pi}{3}$.
Period of $f(x) = \frac{\pi}{2} + \frac{\pi}{3} = \pi$

37) b $\lim_{x\to\infty} \frac{\tan x}{x} = \lim_{x\to\infty} \left(\frac{\sin x}{x}\right) \times \left(\frac{1}{\cos x}\right) = 0 \times (a\ finite\ number) = 0$

38) d $y = \tan^{-1}(\cot x) + \cot^{-1}(\tan x) = \frac{\pi}{2} - \cot^{-1}(\cot x) + \frac{\pi}{2} - \tan^{-1}(\tan x) = \pi - 2x$
$\frac{dy}{dx} = -2$

39) a $\theta = 0^\circ$
$\tan\theta = \frac{dy}{dx}$
$\frac{dy}{dx} = 0$

40) b $\int_{\pi/6}^{\pi/2} \frac{\cos x}{\sin^2 x} = \int_{\pi/6}^{\pi/2} \csc x . \cot x\, dx = |-\csc x|_{\pi/6}^{\pi/2} = -\csc\frac{\pi}{2} + \csc\frac{\pi}{6} = -1 + 2 = 1$

41) a The equation of line which passes through the point (h, k) and cuts off equal intercepts on the axes is:
$x + y = h + k$
$x + y = -2 + 5$
$x + y - 3 = 0$

42) b $\tan\pi = \frac{2\sqrt{3^2 - a\times 7}}{a+7}$
$0 = \frac{2\sqrt{9-7a}}{a+7}$
$a = \frac{9}{7}$

43) b Since, the parabola passes through (3, 2)
$2^2 = 4a \times 3$
$4a = \frac{4}{3}$
Length of latus rectum $= 4a = \frac{4}{3}$

44) b For rectangular hyperbola,
Coeff. of x^2 + Coeff. of y^2 = 0
$5 + \lambda = 0$
$\lambda = -5$

45) a Let the parallel plane be $2x - 3y + z + k = 0$
Since, it passes though (1, -1, 2),
$2 + 3 + 2 + k = 0$
$k = -7$
Hence, the required plane is $2x - 3y + z = 7$

46) a $\tan(180^\circ + \theta) . \tan(90^\circ - \theta) = \tan\theta . \cot\theta = 1$

47) d $\sin^2\theta + 3\cos\theta = 3$
$1 - \cos^2\theta + 3\cos\theta = 3$
$\cos^2\theta - 3\cos\theta + 2 = 0$
$(\cos\theta - 1)(\cos\theta - 2) = 0$
$\cos\theta = 1, \cos\theta = 2$ (not possible)
$\theta = 0$ in $[-\pi, \pi]$

Thus, there is only one solution.

48) c Let, $\sin^{-1} x = \theta$

$\sin\theta = x$

$\therefore \cos 2\theta = \frac{1}{9}$

$1 - 2\sin^2\theta = \frac{1}{9}$

$2\sin^2\theta = \frac{8}{9}$

$x^2 = \frac{4}{9}$

$x = \pm\frac{2}{3}$

49) d Length of side $= \sqrt{3^2+4^2+5^2} = \sqrt{50}$

$\therefore Area = \left(\sqrt{50}\right)^2 = 50$ sq. units

50) d The masses of Cu reacting with a fixed mass of oxygen bear a simple ratio of 1:2.

51) a m cannot be -1 for s-orbital (l=0).

52) b In NO, there is one unpaired electron in antibonding π molecular orbital.

53) c OF_2

$x + 2(-2) = 0$

$x = 2$

54) c Gold has higher reduction potential than iron. Hence, iron is easily oxidized to Fe^{2+}.

55) b Li shows diagonal relationship with Mg and Be shows its relationship with Al.

56) c

57) a Group II metal hydroxides become more soluble in water as you go down the column. This trend can be explained by the decrease in the lattice energy of the hydroxide salt and by the increase in the coordination number of the metal ion as you go down the column.

58) a $2NH_3 + 3CuO \rightarrow N_2 + 3Cu + 3H_2O$

59) c The tendency to donate lone pair of electrons i.e., nucleophilicity increases with the decrease in electronegativity of the atom (F>O>N>C). Thus CH_3^- has the highest nucleophilicity.

60) b As s character is highest in sp-hybridized carbon, the electronegativity of carbon is highest because it lies close to the nucleus and removes electrons from hydrogen, releases ions easily and acts as acid.

<u>Section-B</u> (2 marks)

61) d The first paragraph states that chocolate contains theobromine, which cats and dogs cannot process. It also says that theobromine is "similar to caffeine," but not that chocolate contains caffeine.

62) a This question requires you to make some inferences. The passage states that 20 ounces of milk chocolate can kill a 20-pound dog; it also states that cats can be harmed by smaller amounts than dogs. Therefore, a cat could be poisoned by less than 20 ounces of milk chocolate.

63) d The final paragraph states that dogs like to eat whatever they see their owners eating. This is a form of imitation.

64) b The passage mentions that humans can eat chocolate safely, but cats and dogs cannot. Therefore, one might apply choice b to the topic.

65) c Time taken by the bomb to fall through a height of 490 m is:

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 490}{9.8}} = 10$

Distance at which the bomb strikes the ground = horizontal velocity × time

$= 360\ \text{km/hr} \times 10\ \text{s} = 360\ \text{km/hr} \times \frac{10}{3600}\text{h} = 1\ \text{km}$

66) b Potential energy of a stretched spring, $U = \frac{1}{2}kx^2$

As, $F = kx$

$x = \frac{F}{k}$

$\therefore U = \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{F^2}{2k}$

For same force, $U \propto \frac{1}{k}$

$\frac{U_1}{U_2} = \frac{k_2}{k_1} = \frac{3000}{1500} = 2:1$

67) b $\frac{E_{sphere}}{E_{cylinder}} = \frac{\frac{1}{2}I_s\omega_s^2}{\frac{1}{2}I_c\omega_c^2}$

Here, $I_s = \frac{2}{5}mR^2, I_c = \frac{1}{2}mR^2, \omega_c = 2\omega_s$

$\frac{E_{sphere}}{E_{cylinder}} = \frac{\frac{2}{5}mR^2\times\omega_s^2}{\frac{1}{2}mR^2\times(2\omega_s)^2} = \frac{4}{5}\times\frac{1}{4} = \frac{1}{5}$

68) c Gravitational potential on the surface of the shell is:

$V = \text{Gravitational potential due to particle } (V_1) + \text{Gravitational potential due to shell itself } (V_2)$

$= -\frac{Gm}{R} + \left(-\frac{G(3m)}{R}\right) = -\frac{4Gm}{R}$

69) a Let the radius of bigger drop is R and smaller drop is r, then

$\frac{4}{3}\pi R^3 = 8\times\frac{4}{3}\pi r^3$

$R = 2r$

Terminal velocity, $v \propto r^2$

$\frac{v'}{v} = \left(\frac{R}{r}\right)^2 = \left(\frac{2r}{r}\right)^2 = 4$

$v' = 4\times v = 4\times 8 = 32$ cm/s

70) a $V_T = V_0(1+\gamma\Delta T)$

$\frac{V_T - V_0}{V_0} = \gamma\Delta T$

$\frac{0.24}{100} = \gamma\times 40$

$\gamma = 6\times10^{-5}\ °C^{-1}$

Coefficient of linear expansion,

$\alpha = \frac{\gamma}{3} = \frac{6\times10^{-5}}{3} = 2\times10^{-5}\ °C^{-1}$

71) d For an adiabatic process, $\frac{T^\gamma}{P^{\gamma-1}} = \text{constant}$

$\therefore \left(\frac{T_i}{T_f}\right)^\gamma = \left(\frac{P_i}{P_f}\right)^{\gamma-1}$

$P_f = P_i\left(\frac{T_f}{T_i}\right)^{\frac{\gamma}{\gamma-1}} = 2\left(\frac{927+273}{27+273}\right)^{\frac{1.4}{1.4-1}} = (2)\times(4)^{1.4/0.4} = (2)\times(2)^{7/2} = 2^8 = 256$ atm

72) c Here, $f_A = 258$ Hz, $f_B = 262$ Hz

Let the frequency of unknown tuning fork be f.

It produces f_b beats with A and $2f_b$ with B. Therefore,

$f_A - f = f_b$ --- (1)

$f_B - f = 2f_b$ --- (2)

Subtract (1) from (2), we get,

$f_B - f_A = f_b$

$f_b = 262 - 258 = 4$ Hz

From (1), we get,

$f = f_A - f_b = 258 - 4 = 252$ Hz

73) a Total capacitance in the circuit is:

$C = \frac{3\times6}{3+6} + 2 = 2 + 2 = 4\ \mu F$

Energy $= \frac{1}{2}QV = \frac{1}{2}CV^2$ $(\because Q = CV)$

$= \frac{1}{2}\times4\times2^2 = 8\ \mu J$

74) c Here, $r = 15$ cm $= 15\times10^{-2}$ m, $N = 1500$ turns, $I = 1.2$ A, $\mu_r = 800$

Number of turns/length (n) $= \frac{N}{2\pi r} = \frac{3500}{2\pi\times15\times10^{-2}} = 3715.5$

$B = \mu_0\mu_r nI = 4\pi\times10^{-7}\times800\times3715.5\times1.2 = 4.48$ T

75) b $R = \frac{V}{I_g} - G$

$R = \frac{2}{2\times10^{-3}} - 12 = 1000 - 12 = 988\ \Omega$

76) a $X_C = \frac{1}{2\pi fC} = \frac{1}{2\times3.14\times50\times0.1\times10^{-4}} = 3.2\times10^4\ \Omega$

$Z = \sqrt{R^2 + {X_C}^2} = \sqrt{100 + 10.28\times10^8} = 3.2\times10^4\ \Omega$

$I_{rms} = \frac{E_{rms}}{Z} = \frac{100}{3.2\times10^4} = 3.14\times10^{-3}\ A = 3.14\ mA$

77) d $\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$

According to question, $\delta_m = A$

$\sqrt{3} = \frac{\sin\left(\frac{A+A}{2}\right)}{\sin\frac{A}{2}}$

$\sqrt{3} = \frac{\sin A}{\sin\frac{A}{2}} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}}$

$\cos\frac{A}{2} = \frac{\sqrt{3}}{2} = \cos 30^\circ$

$\frac{A}{2} = 30^\circ$

$A = 60^\circ$

78) b Radius of n^{th} orbit:

$r_n = \frac{a_0 n^2}{Z}$

For hydrogen atom, $Z = 1, n = 1$(for ground state)

$r_1 = a_0$

For Be^{3+}, $Z = 4$

$r_2 = \frac{a_0 n^2}{4}$

According to question,

$r_1 = r_2$

$a_0 = \frac{a_0 n^2}{4}$

$n = 2$

79) d $b^2\sin 2C + c^2\sin 2B = 4R^2 sin^2 B.2\sin C.\cos C + 4R^2 sin^2 C.2\sin B.\cos B$

$= 8R^2\sin B.\sin C(\sin B\ cos\ C + \cos B\sin C)$

$= 8R^2\sin B.\sin C.\sin(B + C)$

$= 8R^2\sin Bsin\ Csin\ A$

$= 8R^2\times\frac{b}{2R}\times\frac{c}{2R}\times\frac{a}{2R} = \frac{abc}{R} = 4\Delta$

80) b $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \sin^{-1}\left(\frac{2b}{1+b^2}\right) = \tan^{-1}x$

$2\tan^{-1}a + 2\tan^{-1}b = 2\tan^{-1}x$

$2\tan^{-1}\left(\frac{a+b}{1-ab}\right) = 2\tan^{-1}x$

$x = \frac{a+b}{1-ab}$

81) c Let x^{-17} occurs in T^{p+1}

$p = \frac{15(4)-(-17)}{4+3} = 11$

$\therefore r = p + 1 = 11 + 1 = 12$

82) d As given, $T_3 = a^2$

$ar^2 = a^2$

$a = r^2$

Now, $T_2 = 8$

$ar = 8$

$r^2.r = 8$

$r^3 = 8$

$r = 2$

$T_8 = ar^7 = 4\times2^7 = 512$

83) d $\begin{vmatrix} 1+\omega & \omega^2 & -\omega \\ 1+\omega^2 & \omega & -\omega^2 \\ \omega+\omega^2 & \omega & -\omega^2 \end{vmatrix} = \begin{vmatrix} -\omega^2 & \omega^2 & -\omega \\ -\omega & \omega & -\omega^2 \\ -1 & \omega & -\omega^2 \end{vmatrix}$ $\because (1+\omega+\omega^2=0)$

Operate $R_3 \to R_3 - R_2$

$= \begin{vmatrix} -\omega^2 & \omega^2 & -\omega \\ -\omega & \omega & -\omega^2 \\ \omega-1 & 0 & 0 \end{vmatrix} = (\omega-1)(-\omega^4+\omega^2) = (\omega-1)(-\omega+\omega^2) = \omega(\omega-1)^2$

$= \omega(\omega^2-2\omega+1) = \omega^3 - 2\omega^2 + \omega = (1+\omega) - 2\omega^2 = -\omega^2 - 2\omega^2 = -3\omega^2$

84) a f(x) is real if $\frac{\pi^2}{9} - x^2 \geq 0$

$x^2 \leq \frac{\pi^2}{9}$

$|x| \leq \frac{\pi}{3}$

Minimum value of $f(x) = 0$ if $x = \frac{\pi}{3}$

Maximum value of $f(x) = \tan\frac{\pi}{3} = \sqrt{3}$ if $x = 0$

$\therefore R_f = [0, \sqrt{3}]$

85) c $\lim\limits_{x\to 1} \frac{ab^x - a^x b}{x-1}$ $\left(\frac{0}{0}\ form\right)$

$= \lim\limits_{x\to 1} \frac{a(b^x.\log b) - (a^x.\log a)b}{x-1}$ $(By\ L-Hospital's\ rule)$

$= ab\log b - ab\log a = ab\ (\log b - \log a) = ab\log\left(\frac{b}{a}\right)$

86) d $\sin y = x\sin(a+y)$

$x = \frac{\sin y}{\sin(a+y)}$

$\frac{dx}{dy} = \frac{\sin(a+y).\cos y - \sin y.\cos(a+y)}{\sin^2(a+y)} = \frac{\sin(a+y-y)}{\sin^2(a+y)}$

$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

87) c $x^2 = 32y \Rightarrow \frac{dy}{dx} = \frac{x}{16}$

$y^2 = 4x \Rightarrow \frac{dy}{dx} = \frac{2}{y}$

At (16, 8), $\left(\frac{dy}{dx}\right)_1 = m_1 = \frac{16}{16} = 1$

$\left(\frac{dy}{dx}\right)_2 = m_2 = \frac{2}{8} = \frac{1}{4}$

Required angle = $\tan^{-1}\left(\frac{1-\frac{1}{4}}{1+1.1/4}\right) = \tan^{-1}\left(\frac{3}{5}\right)$

88) d $\int \frac{dx}{\sqrt{x}(3+x)}$

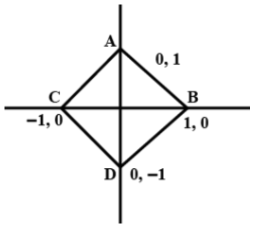
Put $\sqrt{x} = z$

$\frac{1}{2\sqrt{x}}dx = dz$

$dx = 2z\ dz$

$\int \frac{dx}{\sqrt{x}(3+x)} = \int \frac{2z\ dz}{z(z^2+3)} = 2\int \frac{dz}{z^2+3} = \frac{2}{\sqrt{3}}\tan^{-1}\frac{z}{\sqrt{3}} + c = \frac{2}{\sqrt{3}}\tan^{-1}\sqrt{\frac{x}{3}} + c$

89) d



$|x| + |y| = 1$ represent a square with length of each diagonal = 2

$\therefore A = \frac{1}{2} \times 2 \times 2 = 2$

90) a Given equation can be written as:

$3x^2 + 2hxy + (-3)y^2 + 2(-20)x + 2(25)y - 75 = 0$

It will represent a pair of straight lines if
$3(-3)(-75) + 2(15)(-20) \times h - 3(15)^2 + 3(-20)^2 + 75h^2 = 0$
$675 - 600h - 675 + 1200 + 75h^2 = 0$
$h^2 - 8h + 16 = 0$
$(h-4)^2 = 0$
$h = 4, 4$

91) b $C_1 : x^2 + y^2 + 4x + 22y + c = 0$
$C_2 : x^2 + y^2 - 2x + 8y - d = 0$
Centre $(C_2) = (1, -4)$
Since, C_1 bisects the circumference of C_2, the common chord to the circles must be the diameter of second circle.
Equation of common chord is:
$C_1 - C_2 = 0$
i.e.., $6x + 14y + (c + d) = 0$
The centre of second circle (1, -4) lies on it.
$6(1) + 14(-4) + c + d = 0$
$c + d = 50$

92) c As given, $\frac{2b^2}{a} = \frac{1}{2}(2b)$
$2b = a$
$4b^2 = a^2$
$4a^2(1 - e^2) = a^2$
$1 - e^2 = \frac{1}{4}$
$e = \frac{\sqrt{3}}{2}$

93) b As given, $cos^2\frac{\alpha}{2} + cos^2\frac{\beta}{2} + cos^2\frac{\gamma}{2} = 1$
$2cos^2\frac{\alpha}{2} + 2cos^2\frac{\beta}{2} + 2cos^2\frac{\gamma}{2} = 2$
$(1 + \cos\alpha) + (1 + \cos\beta) + (1 + \cos\gamma) = 2$
$\cos\alpha + \cos\beta + \cos\gamma = -1$

94) a $2KOH + CO_2 \rightarrow K_2CO_3 + H_2O$
2(39+16+1) 22.4 L
=102 g
22.4 dm^3 of CO_2 at STP requires 112 g KOH
11.2 dm^3 of CO_2 at STP will require $\frac{112}{22.4} \times 11.2 = 56\ g$ KOH

95) a Equivalent weight of dibasic acid = $\frac{\text{Molecular weight}}{2} = 100$
Strength = 0.1N, mass(m) = ?, V = 100 mL
Normality (N) = $\frac{\text{mass}}{\text{E}} \times \frac{1000}{\text{V(L)}}$
M = $\frac{\text{ENV}}{1000} = \frac{100\times100\times0.1}{1000} = 1\text{g}$

96) d $\text{CaF}_2 \rightleftharpoons \text{Ca}^{2+} + 2\text{F}^-$
$2 \times 10^{-4}\text{M}$ $2 \times 10^{-4}\text{M}$ $2 \times 2 \times 10^{-4}\text{M}$
K_{sp} of CaF_2 = $[\text{Ca}^{2+}][\text{F}^-]^2$
= $[2 \times 10^{-4}][4 \times 10^{-4}]^2$
= $32 \times 10^{-12}(\text{mol/L})^2$

97) b

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  5       4      3      2   1
CH3 – CH – CH = C – CHO
          |          |
         OH        CH3
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98) c At NTP, 1 mole of H_2 = 2 g
22400 mL of H_2 = 2 g
112 mL of H_2 = $\frac{2}{22400} \times 112 = 0.01$ g
1 F of electricity displace 1 g of H_2

0.01 g of hydrogen is displaced by 0.01 F of electricity
1 F can deposit 108 g of Ag
0.01 F can deposit 0.01×108 = 1.08 g of Ag

99) a The inductive effect of the alkyl group on a saturated carbon chain follows the order:
$(CH_3)_3C- > (CH_3)_2CH- > CH_3CH_2-$
However, when an alkyl group is attached to an unsaturated system such as double bond or a benzene ring, the order of inductive effect is actually reversed. This effect is called hyperconjugation effect or Baker-Nathan effect. Now, the reactivity order is: $(CH_3)_3C- < (CH_3)_2CH- < CH_3CH_2-$

100) c Smaller the size of the ion, more is the hydration and lesser is the mobility.
So, the order of mobility will be:
$Rb^+ > K^+ > Na^+ > Li^+$

Thank You!!!!!!