INSTITUTE OF ENGINEERING

Model Entrance Exam

(Set-8 Solutions)

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Date: 2080/04/06
(July-22)

Duration: 2 hours
Time: 8 AM – 10 AM

<u>Section-A</u> (1 marks)

1) a
2) a
3) b
4) b
5) d
6) b
7) c
8) d
9) d
10) a
11) c
12) c
13) b $F = Bil\sin\theta$

$B = \frac{F}{il\sin\theta} = \frac{[MLT^{-2}]}{[A][L]} = [MT^{-2}A^{-1}]$

14) b
15) b At highest point, horizontal component of velocity only acts and acceleration due to gravity is vertically downward.
16) b $a = \omega^2 R$

$\omega = \sqrt{\frac{a}{r}} = \sqrt{\frac{9.8}{0.2}} = \sqrt{49} = 7$ rad/s

17) b On inclination, vertical height h of liquid remains same.

$l\,cos\alpha = h, cos\alpha < l$

$\therefore l > h$

18) c
19) c
20) c It does not intercept the earth's magnetic lines of force. Hence, no emf will be induced in it.
21) c
22) a
23) b
24) c
25) a
26) a $C = \sqrt{\frac{3RT}{M}} \propto \sqrt{T}$

$C = \sqrt{\frac{927+273}{27+273}}.C_0 = \sqrt{4}\,C_0 = 2C_0$

27) d $\frac{C}{5} = \frac{F-32}{9}$

$-\frac{235}{5} = \frac{F-32}{9}$

$F = -391^\circ F$

28) c The time period of a simple pendulum is:

$T = 2\pi\sqrt{\frac{l}{g}}$

On squaring both sides,

$T^2 = \frac{4\pi^2 l}{g}$

$T^2 \propto l$

So, the graph between time period T and length l of a pendulum is a parabola.

29) c When polarity of the battery is reversed, the P-N junction becomes reverse biased so no current flows.
30) c $\lim_{x\to\infty} x\cos\left(\frac{\pi}{4x}\right).\sin\left(\frac{x}{4\pi}\right) = \lim_{x\to\infty}\cos\left(\frac{\pi}{4x}\right).\sin\left(\frac{\frac{\pi}{4x}}{\frac{\pi}{4x}}\right).\frac{\pi}{4} = 1.1.\frac{\pi}{4} = \frac{\pi}{4}$
31) d $y = \tan^{-1}(\cot x)$

$y = \tan^{-1}\left(\tan\left(\frac{\pi}{2} - x\right)\right) = \frac{\pi}{2} - x$

$\frac{dy}{dx} = 0 - 1 = -1$

32) d $f(x) = 2x^3 - 9x^2 + 12x - 20$

$f'(x) = 6x^2 - 18x + 12 = 6(x-1)(x-2)$
For $1 < x < 2, f'(x) < 0$
Thus, $f(x)$ is decreasing in (1,2).

33) d $\int \frac{dx}{x(1+\log x)} = \int \frac{1/x}{1+\log x} dx = \log(1+\log x) + c$

34) b $\int_0^1 \left(\sin^{-1} \frac{2x}{1+x^2} + 2\cot^{-1} x\right) dx = \int_0^1 (2\tan^{-1} x + 2\cot^{-1} x)\, dx = 2\int_0^1 \frac{\pi}{2} dx = 2.\frac{\pi}{2} = \pi$

35) b $f(2) = 3$
$3(2)^2 - 2(2) + k = 3$
$k = -5$

36) c $y = 1 + \frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!} + \cdots$
$y = e^{x^2}$
$\log y = x^2$
$x = \sqrt{\log y}$

37) a $(1+i)^6 + (1-i)^6 = ((1+i)^2)^3 + ((1-i)^2)^3 = (2i)^3 + (-2i)^3 = 8i^3 - 8i^3 = 0$

38) b $A = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$
Adj. A $= \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}^T = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

39) d To form a parallelogram, we require two parallel lines from each set. Hence, number of parallelograms = ${}^4C_2 \times {}^3C_2 = 18$

40) c $f(x) = 2\sin x, g(x) = \cos^2 x$
$(f+g)\frac{\pi}{3} = f\left(\frac{\pi}{3}\right) + g\left(\frac{\pi}{3}\right) = 2\sin\frac{\pi}{3} + \cos^2\left(\frac{\pi}{3}\right) = \sqrt{3} + \frac{1}{4}$

41) b $3^{2n} - 1 = (3^2)^n - 1 = (1+8)^n - 1$ which is a multiple of 8.
Hence, $A \subset B$.

42) a When $x = 1, y = 1^2 + 2 \times 1 = 3$
When $x = 3, y = 3^2 + 2 \times 3 = 15$
So, slope of line joining these points $= \frac{15-3}{3-1} = 6$

43) b Given equation can be written as:
$(3x+y)^2 - (2)^2 = 0$
$(3x+y+2)(3x+y-2) = 0$
$3x+y+2 = 0, 3x+y-2 = 0$
Which are parallel but not coincident.

44) b $4x + 3y = 5$
$y = -\frac{4}{3}x + \frac{5}{3}$
Note:
The length of the intercept cut off from the line $y = mx + c$ by the circle $x^2 + y^2 = a^2$ is:
$d = 2\sqrt{a^2 - \frac{c^2}{1+m^2}}$
$\therefore$ Required length $= 2\sqrt{26 - \frac{25/9}{1+16/9}} = 2\sqrt{26-1} = 2 \times 5 = 10$ units

45) c Given, $2x + y + \lambda = 0$
$y = -2x - \lambda$
Using condition of normal,
$c = -2am - am^3$
$-\lambda = -2(-2)(-2) + 2(-2)^3$
$-\lambda = -24$
$\lambda = 24$

46) a $9y^2 - 4x^2 = 36$
$\frac{x^2}{9} - \frac{y^2}{4} = -1$
L.R. $= \frac{2a^2}{b} = \frac{18}{2} = 9$

47) b Since, $z = 0$ represents the xy-plane. So, $z = c$ represents a plane parallel to xy-plane.

48) b $12\sin\theta - 9\sin^2\theta = -[9\sin^2\theta - 12\sin\theta] = -[(3\sin\theta)^2 - 2.3\sin\theta\,.\,2 + 2^2] + 2^2 = -(3\sin\theta - 2)^2 + 4$

$\therefore$ Maximum value of $12\sin\theta - 9\sin^2\theta = 4$

49) d $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\left(2\pi - \frac{5\pi}{6}\right)\right) = \cos^{-1}\left(\cos\frac{5\pi}{6}\right) = \frac{5\pi}{6}$

50) a 22.4 L of O_2 at NTP = 32 g

1 L of O_2 at NTP = $\frac{32}{22.4} = 1.43$ g

51) b Principal quantum number (n) describes size of orbital, average distance of nucleus and total energy of the electron.
Azimuthal quantum number (l) gives the orbit, sub-shells or sub-energy level in which the electron is located.
Magnetic quantum number (m) describes the orientation of sub-shells.
Spin quantum number (s) describes the spin of electrons.

52) b Conditions of H-bonding:
i) The molecule must possess a higher electronegative atom such as F, O or N directly linked to H-atom.
ii) Size of electronegative atom should be small.

53) d Since the charge of the ion is -2, so, the sum of oxidation number of all individual atoms is equal to -2.

$2x + 7(-2) = -2$

$2x = 12$

$x = 6$

54) b In a group, electronegativity decreases from top to bottom. In a period, electronegativity increases from left to right.

55) a The function of a salt bridge in an electrolytic cell is to maintain electrical neutrality in solution and prevent voltage drop. Once the salt bridge is removed, the voltage drops to zero and current will stop flowing in the circuit of the cell.

56) d

57) b During smelting

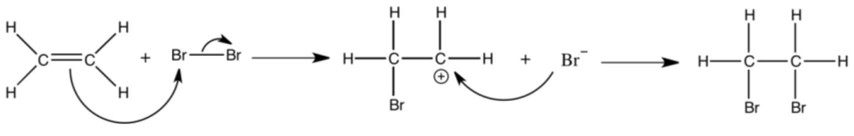
$2\,FeS + 3\,O_2 \rightarrow 2\,FeO + 2\,SO_2\uparrow$

$FeO + SiO_2 \rightarrow FeSiO_3$ (slag)

58) c $CuSO_4.5H_2O$: Blue vitriol

$FeSO_4.7H_2O$: Green vitriol

59) a



60) c $C_2H_5 - O - C_2H_5$ and $CH_3 - O - C_3H_7$ are metamers. So, the type of isomerism is metamerism.

<u>Section-B</u> (2 marks)

61) d The first two sentences of the passage indicate that a backdraft is dangerous because it is an explosion. The other choices are dangers, but they do not define a backdraft.

62) b The second paragraph indicates that there is little or no visible flame with a potential backdraft. The other choices are listed at the end of the second paragraph as warning signs of a potential backdraft.

63) c This is stated in the last paragraph. Choice a is not mentioned in the passage. The other choices would be useless or harmful.

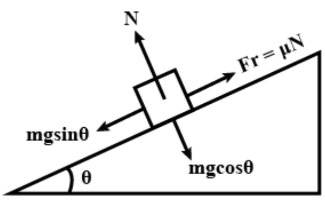
64) a The passage indicates that hot, smoldering fires have little or no visible flame and insufficient oxygen. It can reasonably be inferred, then, that more oxygen would produce more visible flames.

65) c $h = \frac{1}{2}gT^2$

$h' = \frac{1}{2}g\left(\frac{T}{3}\right)^2$

Height from ground $= h - h' = h = \frac{1}{2}gT^2 - \frac{1}{2}g\left(\frac{T}{3}\right)^2 = \frac{1}{2}gT^2\left(1 - \frac{1}{9}\right)h = \frac{8}{9}h$

66) c



$mg\sin\theta - f = ma$

$mg\sin\theta - \mu_k mg\cos\theta = ma$

$a = g\sin\theta - \mu_k g\cos\theta = (\sin\theta - \mu_k\cos\theta)g = \left(\frac{\sqrt{3}}{2} - 0.25 \times \frac{1}{2}\right) \times 10 = 7.4\ m/s^2$

67) b $K.E. = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)(2\pi n)^2 = \pi^2 MR^2 n^2 = (3.14)^2 \times 72 \times (0.50)^2 \times \left(\frac{72}{60}\right)^2 = 242\ J$

68) a $v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G\times\frac{4}{3}\pi R^3\rho}{R}} = \sqrt{\frac{8}{3}\ G\pi R^2\rho} = R\sqrt{\frac{8}{3}\ G\pi\rho} \propto R$

$\frac{v_p}{v_e} = \frac{R_p}{R_e} = 2$

$v_p = 2v_e = 22\ km/second$

69) c Weight of sphere = Weight of liquid displaced

$V\rho g = \frac{V}{3} \times 13.5\ g + \frac{2V}{3} \times 1.2\ g$

$\rho = \frac{13.5+1.2}{3} = \frac{15.9}{3} = 5.3$

70) c Condition of no difference in length of two rods with rise of temperature is:

$L_1\alpha_1 = L_2\alpha_2$

$L_2 = \frac{L_1\alpha_1}{\alpha_2} = \frac{16\times18\times10^{-6}}{12\times10^{-6}} = 24$ cm

71) b In a closed vessel, $T = constant$

$\frac{P_1}{T_1} = \frac{P_2}{T_2}$

$\frac{P_1}{T_1} = \frac{\left(P_1 + \frac{0.5}{100}P_1\right)}{T_1+2}$

$\frac{T_1+2}{T_1} = 1 + \frac{0.5}{100}$

$1 + \frac{2}{T_1} = 1 + \frac{0.5}{100}$

$\frac{2}{T_1} = \frac{0.5}{100}$

$T_1 = 400\ K = 127^\circ C$

72) b First overtone of closed pipe $P_1 = 3\left(\frac{v}{4l_1}\right)$

Second overtone of open pipe $P_2 = 3\left(\frac{v}{2l_2}\right)$

They are in resonance. So,

$3\left(\frac{v}{4l_1}\right) = 3\left(\frac{v}{2l_2}\right)$

$\frac{l_1}{l_2} = \frac{1}{2}$

73) c $\mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin(90^\circ - i)} = \tan i$ $[\because i + r = 90^\circ]$

$\tan i = \sqrt{3}$

$i = 60^\circ$

74) b The position of n^{th} minima in the diffraction pattern is:

$x_n = \frac{n\lambda D}{d}$

$x_3 - x_1 = (3-1)\frac{\lambda D}{d} = \frac{2\lambda D}{d}$

$d = \frac{2\lambda D}{x_3 - x_1} = \frac{2\times0.50\times6000\times10^{-10}}{3\times10^{-3}} = 2\times10^{-4}$ m

75) d $r = \frac{d}{2} = \frac{4.4}{2} = 2.2\ m$

Charge on the sphere, $q = \sigma \times 4\pi r^2 = 60\times10^{-6} \times 4 \times \frac{22}{7} \times (2.2)^2 = 3.7\times10^{-3}$ C

76) d 6 Ω and 6 Ω are in series, so effective resistance is 12 Ω which is in parallel with 3 Ω, so

$\frac{1}{R} = \frac{1}{3} + \frac{1}{12}$

$R = \frac{36}{15}$

$\therefore I = \frac{V}{R} = \frac{4.8\times15}{36} = 2$ A

77) a Force per unit length, $\frac{F}{l} = \frac{\mu_0}{4\pi}.\frac{2i_1 i_2}{a}$

$F=\frac{\mu_0}{4\pi}.\frac{2i_1i_2}{a}\times l=10^{-7}\times\frac{2\times10\times2}{(10\times10^{-2})}\times 2=8\times10^{-5}$ N

78) c For Balmer series, $n_1=2, n_2=3$ for 1st line and $n_2=4$ for second line.

$\frac{\lambda_1}{\lambda_2}=\frac{\left(\frac{1}{2^2}-\frac{1}{4^2}\right)}{\left(\frac{1}{2^2}-\frac{1}{3^2}\right)}=\frac{3/16}{5/36}=\frac{27}{20}$

$\lambda_2=\frac{20}{27}\lambda_1=\frac{20}{27}\times 6561=4860$ Å

79) b $CaC_2\xrightarrow{H_2O}HC\equiv CH\xrightarrow{H_2SO_4,HgSO_4}CH_3CHO\xrightarrow{LiAlH_4}CH_3CH_2OH$

80) c

81) c Molarity $=\frac{No.of\ moles}{Volume\ in\ litre}=\frac{\frac{2}{40}}{\frac{500}{1000}}=\frac{1}{10}$

$NaOH\rightleftharpoons Na^+ + OH^-$

1 1 1

0.1 M. 0.1 M 0.1 M

$p^{OH}=-\log[OH^-]=-\log[0.1]=-\log[10^{-1}]=1$

$p^{H}=14-p^{OH}=14-1=13$

82) c $E_{metal}=\frac{W_{metal}}{W_{oxygen}}\times 8=\frac{70}{30}\times 8=18.66$

Molecular weight of metal chloride $=\frac{0.72}{100}\times 22400=161.28$

Valency of metal $=\frac{\text{Molecular weight of metal chloride}}{Equivalent\ weight\ of\ metal\ chloride}=\frac{161.28}{18.66+35.5}=3$

MCl_3 is a formula of metal chloride.

83) d Mg lies above Cu in electrochemical series and hence Cu acts as cathode.

$E^0{}_{cell}=E^0{}_{Cu^{2+}/Cu}-E^0{}_{Mg^{2+}/Mg}$

$2.70=0.34-E^0{}_{Mg^{2+}/Mg}$

$E^0{}_{Mg^{2+}/Mg}=0.34-2.70=-2.36\,V$

84) d In NH_3, charge from three bonds was moving towards nitrogen in the direction of lone pairs. In NF_3 and BF_3, due to structure, pyramidal and trigonal planar, NF_3 has more dipole moment. The resultant dipole for BF_3 is zero.

Hence, the sequence is:

$NH_3>NF_3>BF_3$

85) b $CaCO_3\xrightarrow{\Delta}CaO+CO_2$

X Residue

$CaO+H_2O\rightarrow Ca(OH)_2$

Y

$Ca(OH)_2+CO_2+H_2O\rightarrow Ca(HCO_3)_2$

Z

$Ca(HCO_3)_2\xrightarrow{\Delta}CaCO_3+CO_2+H_2O$

X

86) c $\left|\vec{a}+\vec{b}+\vec{c}\right|^2=a^2+b^2+c^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=a^2+a^2+a^2+2(0)=3a^2$

$\therefore\left|\vec{a}+\vec{b}+\vec{c}\right|=\sqrt{3}a$

87) a $P+Q=\frac{\pi}{2}$

$\frac{P}{2}+\frac{Q}{2}=\frac{\pi}{4}$

$\tan\left(\frac{P}{2}+\frac{Q}{2}\right)=\tan\frac{\pi}{4}$

$\frac{\tan\frac{P}{2}+\tan\frac{Q}{2}}{1-\tan\frac{P}{2}.\tan\frac{Q}{2}}=1$

$\frac{-\frac{b}{a}}{1-\frac{c}{a}}=1$

$-b=a-c$

$a+b=c$

88) a $\lim_{x\to0}\frac{\tan x-\sin x}{x^3}=\lim_{x\to0}\frac{\sin x(1-\cos x)}{x^3\cos x}=\lim_{x\to0}\frac{sinx}{x}\times\frac{1-\cos x}{x^2}\times\frac{1}{\cos x}$

$=1\times\lim_{x\to0}\frac{1-\cos x}{x^2}\times\frac{1}{1}$

$=\lim_{x\to0}\frac{\sin x}{2x}$ [By L'Hospital rule]

$=\lim_{x\to0}\frac{\cos x}{2}$ [By L'Hospital rule]

$= \frac{1}{2}$

89) c $y = \log\left(\frac{1-x^2}{1+x^2}\right) = \log(1-x^2) - \log(1+x^2)$

$\frac{dy}{dx} = \frac{1}{1-x^2}.(-2x) - \frac{1}{1+x^2}.(2x) = (-2x)\frac{(1+x^2+1-x^2)}{1-x^4} = \frac{-4x}{1-x^4}$

90) b $y^2 = x$ ------ (1)

Diff. both sides w.r.t. x, we get,

$2y\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y}$

$\tan\frac{\pi}{4} = \frac{1}{2y}$

$y = \frac{1}{2}$

When $y = \frac{1}{2}$, then from (1),

$x = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

∴ Required point = $\left(\frac{1}{4}, \frac{1}{2}\right)$

91) b $\int \frac{\cot\sqrt{x}}{2\sqrt{x}}dx$

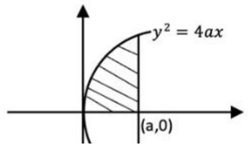
Put $\sqrt{x} = z$

$\frac{dz}{dx} = \frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$

$dz = \frac{1}{2\sqrt{x}}dx$

$\therefore \int \frac{\cot\sqrt{x}}{2\sqrt{x}}dx = \int \cot z\, dz = \log|\sin z| + c = \log\left|\sin\sqrt{x}\right| + c$

92) b



$A = \int_0^a \sqrt{4ax}\, dx = \frac{4}{3}a^2$

93) c $T_{r+1} = {}^{10}C_r\,(\sqrt{x})^{10-r}.\left(\frac{-k}{2}\right)^r = {}^{10}C_r\,(-k)^r.x^{\frac{10-5r}{2}}$

Which does not contain x if

$\frac{10-5r}{2} = 0$

$r = 2$

$\therefore T_{2+1} = 405$

$\Rightarrow {}^{10}C_2\,(-k)^2 = 405$

$k^2 = 9$

$k = \pm 3$

94) b 30, 24, 20 are in HP

$\therefore \frac{1}{30}, \frac{1}{24}, \frac{1}{20}$ are in AP

Common difference of A.P. = $\frac{1}{24} - \frac{1}{30} = \frac{1}{120}$

∴ Next term of AP = $\frac{1}{20} + \frac{1}{120} = \frac{7}{120}$

Thus, next term of HP = $\frac{120}{7}$

95) d $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 2I$

$A^5 = 2^5 I = 32I = 16(2I) = 16A$

96) a Since, $-1 \le \cos 3x \le 1$

$1 \ge -\cos 3x \ge -1$

$3 \ge 2 - \cos 3x \ge 1$

$\frac{1}{3} \le \frac{1}{2-\cos 3x} \le 1$

∴ Range of f is $\left[\frac{1}{3}, 1\right]$

97) a Given lines are:

$3x - 4y + 10 = 0$

$-5x + 12y + 10 = 0$

Now equation of required bisector is:

$\frac{3x-4y+10}{\sqrt{9+16}} = -\frac{-5x+12y+10}{\sqrt{25+114}}$

$13(3x - 4y + 10) + 5(-5x + 12y + 10) = 0$

$7x + 4y + 90 = 0$

98) b Equation of common tangents at their point of contact is:

$S_1 - S_2 = 0$

$x^2 + y^2 + 2x - 8y + 8 - x^2 - y^2 - 10x + 2y - 22 = 0$

$-8x - 6y - 14 = 0$

$4x + 3y + 7 = 0$

99) c Given, $ae = 1, a = 2$

$e = \frac{1}{2}$

Also, $ae = \sqrt{a^2 - b^2}$

$1 = 4 - b^2$

$b^2 = 3$

$b = \sqrt{3}$

∴ Minor axis $= 2b = 2\sqrt{3}$

100) a Dr's of CD are: 3-1, 5-2, 7-3 i.e., 2, 3, 4

∴ Projection $= \frac{1}{\sqrt{4+9+16}}[2(3-2) + 3(5-3) + 4(-3+1)] = \frac{1}{\sqrt{29}}(2 + 4 - 6) = 0$

Thank You!!!!!!