INSTITUTE OF ENGINEERING

Model Entrance Exam

(Set-9 Solutions)

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Date: 2080/04/13
(July-29)

Duration: 2 hours
Time: 8:30 AM – 10:30 AM

<u>Section-A</u> (1 marks)

1) c
2) d
3) d
4) b
5) a
6) c
7) d
8) c
9) a
10) a
11) a
12) c
13) b $\lim_{x\to 1}(1-x)\tan\frac{\pi x}{2} = \lim_{x\to 1}\frac{1-x}{\cot\frac{\pi x}{2}}\left(\frac{0}{0}\text{ form}\right)$
$= \lim_{x\to 1}\frac{-1}{-\frac{\pi}{2}\operatorname{cosec}^2\frac{\pi x}{2}} = \frac{-1}{-\frac{\pi}{2}\times 1} = \frac{2}{\pi}$
14) d $\frac{d}{dx}\left(\frac{\cos x}{\sin x+1}\right) = \frac{(\sin x+1).(-\sin x)-\cos x.\cos x}{(\sin x+1)^2} = \frac{-sin^2x-\sin x-cos^2x}{(\sin x+1)^2} = \frac{-(sinx+1)}{(\sin x+1)^2} = \frac{-1}{\sin x+1}$
15) d $xy = 4 \Rightarrow y = \frac{4}{x}$
$x + 16y = x + \frac{64}{x} = x + \frac{8^2}{x}$
Note:
The function $f(x) = x + \frac{a^2}{x}$ $(a > 0)$ has:
i) local maxima at $x = -a$ and
ii) local minima at $x = +a$

Here, maxima occur at $x = -8$.
Hence, maximum value = $-8 - \frac{64}{8} = -16$
16) c $\int\frac{dx}{x+\sqrt{x}} = \frac{dx}{\sqrt{x}(\sqrt{x}+1)} = 2\int\frac{\frac{1}{2\sqrt{x}}dx}{\sqrt{x}+1} = 2\log(\sqrt{x}+1) + c$
17) c Let us assume roots as 2 and 3.
$x^2 - (2+3)x + 2\times 3 = 0$
$x^2 - 5x + 6 = 0$
Comparing it with the given equation, we get: $k = 5$.
18) c $\sum_{n=0}^{\infty}\frac{(log_e x)^{2n}}{(2n)!} = 1 + \frac{(log_e x)^2}{2!} + \frac{(log_e x)^4}{4!} + \cdots + \infty = \frac{e^{\log x}+e^{-\log x}}{2} = \frac{x+x^{-1}}{2}$
19) c Let r be the common ratio
$405 = 5r^4$
$r^4 = 81$
$r = 3$
$\therefore z = \frac{405}{3} = 135$
20) c Replace i by -i,
$\therefore$ Required complex number $= \frac{1}{-i-1} = \frac{-1}{i+1}$
21) b $A - B$ exists when A and B are of the same order. So, $p = r, q = s$.
22) b He fails if he fail in atleast one of given 5 subject.
So, total number of ways $= 2^5 - 1 = 31$
23) d $fog(x) = f(g(x)) = f\left(\frac{x}{1-x}\right) = \frac{\frac{x}{1-x}}{\frac{x}{1-x}+1} = \frac{x}{x+1-x} = x$
24) a Adding given equations, we get:
$(a+b+c)x + (a+b+c)y + (a+b+c) = 0$

$0.x + 0.y + o = 0 \qquad (\because a + b + c = 0)$
Thus, lines are concurrent.

25) c Given equations can be written as:
$x^2 + 2(2)xy + ky^2 = 0$
It represents coincident lines if $h^2 - ab = 0$
$2^2 - 1 \times k = 0$
$k = 4$

26) b The line $y = mx + c$ intersects the parabola $y^2 = 4ax$ in
i) two real points if $\frac{mc}{a} < 1$ (chord)
ii) one single point if $\frac{mc}{a} = 1$ (tangent)
iii) two imaginary points if $\frac{mc}{a} > 1$ (outside)

27) a $16x^2 + 25y^2 = 400$
$\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$
Here, $a > b$
So, vertices $= (\pm a, 0) = (\pm 5, 0)$

28) d Equation of plane passing through the point (x_1, y_1, z_1) and making equal intercepts on the coordinate axes is:
$x + y + z = x_1 + y_1 + z_1$
Here, $x + y + z = 1 - 1 + 2$
$\therefore x + y + z = 2$

29) d $4 \sin A \, cos^3 A - 4 \cos A \, sin^3 A = 4 \cos A \sin A \, (cos^2 A - sin^2 A) = 2(2 \sin A \cos A)(\cos 2A)$
$= 2(\sin 2A)(\cos 2A) = \sin 4A$

30) d $sin^2\theta + 3 \cos\theta = 3$
$1 - cos^2\theta + 3\cos\theta = 3$
$cos^2\theta - 3\cos\theta + 2 = 0$
$(\cos\theta - 1)(\cos\theta - 2) = 0$
$\cos\theta = 1, \cos\theta = 2$ (not possible)
$\theta = 0$ in $[-\pi, \pi]$
Thus, there is only one solution.

31) a For $x = -\frac{1}{2}$, $\sin^{-1}\left(\frac{1}{x}\right) = \sin^{-1}(2)$, which is not defined.

32) c $\vec{a}.\vec{b} \geq 0$
$ab \cos\theta \geq 0$
$\cos\theta \geq 0$
$0 \leq \theta \leq \frac{\pi}{2}$

33) a In this reaction $C_2O_4{}^{2-}$ acts as a reducing agent because it reduces Mn from +7 to +2.

34) a
35) b
36) d
37) d
38) d
39) a
40) d
41) c

42) a The sequence of orbital in order of increasing energy are: 1s, 2s, 2p, 3s, 3p, 4s, 3d, 4p, 5s …… etc. So, after filling the electrons in np, electrons are filled in $(n + 1)s$.

43) d Normality $= \frac{no.of\ gram\ equivalent}{volume\ of\ solution\ in\ litres} = \frac{m}{eq.wt.\times V} = \frac{6.36\times 1000}{49\times 600} = 0.2\ N$

44) a $[RC] = \left[\frac{V}{I} \times \frac{q}{V}\right] = \left[\frac{q}{I}\right] = \left[\frac{charge}{current}\right] = [time] = T$

45) b
46) d
47) a
48) d Rise of liquid in a capillary tube is due to surface tension.

49) c Water cools to 0°C so ice and water are at the same temperature. Water cannot give out its latent heat to ice and cannot freeze.

50) d

51) c If incident angle of light beam is more than the critical angle, total internal reflection occurs.

52) b Magnifying power $\propto \frac{1}{f_e} \propto P_e$

If power of eye lens is more, then magnifying power is more.

53) c

54) b A charged conductor has the same potential (V) at all points whatever its shape. So, along its surface $dV = 0, E = \frac{dV}{dr} = 0$, along its surface i.e., the component of E at a point on the surface is zero. Hence, E is normal to the surface at the point.

55) c In series, current remains same in each component.

56) c Steady magnetic field has no effect on magnitude of velocity of charged particle.

57) a

58) c

59) d Torque due to central force is zero.

$\tau = \frac{dL}{dt} = 0$

$L = constant$

60) a Since, superconducting material and liquid nitrogen both are diamagnetic in nature, the dipped ball of superconductor in liquid nitrogen also behaves as a diamagnetic material. When it is placed near a magnet, it will be repelled.

Section-B (2 marks)

61) d The second sentence of paragraph 1 states that probes record responses. Paragraph 2 says that electrodes accumulate much data.

62) c The tone throughout the passage suggests the potential for microprobes. They can be permanently implanted, they have advantages over electrodes, they are promising candidates for neural prostheses, they will have great accuracy, and they are flexible.

63) d According to the third paragraph, people who lack biochemicals could receive doses via prostheses. However, there is no suggestion that removing biochemicals would be viable.

64) a The first sentence of the third paragraph says that microprobes have channels that open the way for delivery of drugs. Studying the brain (choice d) is not the initial function of channels, though it is one of the uses of the probes themselves.

65) d By sine formula

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{6}{\sin 30^\circ} = \frac{8}{\sin B}$

$\sin B = \frac{4}{6} = \frac{2}{3}$

$B = \sin^{-1}\frac{2}{3}$

$\sin^{-1} x = \sin^{-1}\frac{2}{3}$

$x = \frac{2}{3}$

66) a As given,

$sin^2\theta = \sin\phi . \cos\phi$

$2sin^2\theta = \sin 2\phi$

$1 - 2sin^2\theta = 1 - \sin 2\phi$

$\cos 2\theta = 1 - \cos\left(\frac{\pi}{2} - 2\phi\right)$

$\therefore \cos 2\theta = 2sin^2\left(\frac{\pi}{4} - \phi\right)$

67) b $\lim_{x\to\frac{\pi}{2}}\left[x\tan x - \left(\frac{\pi}{2}\right)\sec x\right] = \lim_{x\to\frac{\pi}{2}}\left[\frac{x\sin x}{\cos x} - \frac{\pi}{2\cos x}\right] = \lim_{x\to\frac{\pi}{2}}\left[\frac{x\sin x}{\cos x} - \frac{\pi}{2\cos x}\right] = \lim_{x\to\frac{\pi}{2}}\frac{2x\sin x - \pi}{2\cos x}$ $\left(\frac{0}{0}\text{ form}\right)$

$= \lim_{x\to\frac{\pi}{2}}\frac{[2\sin x + 2x\cos x]}{-2\sin x}$ (Applying L'Hospital Rule)

$= \frac{2(1)+2.\frac{\pi}{2}.0}{-2(1)} = \frac{2}{-2} = -1$

68) b $f(x) = \frac{x-|x-1|}{x} = \begin{cases} \frac{x+x-1}{x}, & x < 1, x \neq 0 \\ \frac{x-(x-1)}{x}, & x \geq 1 \end{cases} = \begin{cases} \frac{2x-1}{x}, & x < 1, x \neq 0 \\ \frac{1}{x}, & x \geq 1 \end{cases}$

Clearly, $f(x)$ is discontinuous at $x = 0$ as it is not defined at $x = 0$. Since $f(x)$ is not defined at $x = 0$, $f(x)$ cannot be differentiable at $x = 0$.

And, $f(x)$ is continuous at $x = 1$, but it is not differentiable at $x = 1$, because $Lf'(1) = 1$ and $Rf'(1) = -1$.

69) d $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{x}(3-x)}{1-3x}\right)\right]$

Put $\sqrt{x} = \tan\theta$ or $\theta = \tan^{-1} x$

$\frac{d}{dx}\left[\tan^{-1}\left(\frac{\tan\theta(3-tan^2\theta)}{1-3tan^2\theta}\right)\right] = \frac{d}{dx}\left[\tan^{-1}\left(\frac{3\tan\theta - tan^3\theta}{1-3tan^2\theta}\right)\right] = \frac{d}{dx}[\tan^{-1}(\tan 3\theta)] = \frac{d}{dx}(3\theta)$

$= \frac{d}{dx}\left(3\tan^{-1}\sqrt{x}\right) = 3\frac{d(\tan^{-1}\sqrt{x})}{\sqrt{x}}.\frac{d\sqrt{x}}{x} = 3.\frac{1}{1+x}.\frac{1}{2\sqrt{x}} = \frac{3}{2\sqrt{x}(1+x)}$

70) b $4x^2 + 9y^2 = 72$

Diff. w.r.t. x, we have

$8x + 18y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{4}{9}\frac{x}{y}$

At (3, 2), $\left(\frac{dy}{dx}\right)_1 = -\frac{4}{9}.\frac{3}{2} = -\frac{2}{3}$

Also, $x^2 - y^2 = 5$

Diff. w.r.t. x, we have

$2x - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x}{y}$

At (3, 2), $\left(\frac{dy}{dx}\right)_2 = \frac{3}{2}$

Since, $\left(\frac{dy}{dx}\right)_1 \times \left(\frac{dy}{dx}\right)_2 = -1$, the curves cut orthogonally.

71) b $\int_{\sqrt{2}}^{x}\frac{dx}{x\sqrt{x^2-1}} = \frac{\pi}{12}$

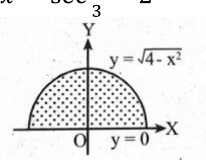
$|\sec^{-1} x|_{\sqrt{2}}^{x} = \frac{\pi}{12}$

$\sec^{-1} x - \frac{\pi}{4} = \frac{\pi}{12}$

$\sec^{-1} x = \frac{\pi}{3}$

$x = \sec\frac{\pi}{3} = 2$

72) a



2 (radius)

Area of the semi-circle $= \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(2^2) = 2\pi$

73) b $T_{r+1} = {}^9C_r\, 3^{9-r}(ax)^r = {}^9C_r\, 3^{9-r}\, a^r x^r$

Coefficient of x^r = ${}^9C_r\, 3^{9-r} a^r$

As given,

Coefficient of x^2 = Coefficient of x^3

${}^9C_2\, 3^{9-2}a^2$ = ${}^9C_2\, 3^{9-3}a^3$

${}^9C_2\, 3^7 a^2$ = ${}^9C_2\, 3^6 a^3$

$a = \frac{9}{7}$

74) d 7^{th} and 25^{th} terms of corresponding A.P. are 10 and 25.

$a + 6d = 10$ and $a + 11d = 25$

On solving,

$a = -8, d = 3$

$T_{20} = \frac{1}{a+19d} = \frac{1}{-8+19\times 3} = \frac{1}{49}$

75) c $z = \frac{-2}{1+\sqrt{3}i} = \frac{-2}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i} = \frac{-2+2\sqrt{3}i}{1-3i^2} = \frac{-2+2\sqrt{3}i}{4} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$

$\therefore Arg(z) = \frac{2\pi}{3}$

76) b For real $f(x)$,
$5x - 3 - 2x^2 \geq 0$
$2x^2 - 5x + 3 \leq 0$
$(2x - 3)(x - 1) \leq 0$
$x \leq \frac{3}{2}$ and $x \geq 1$
$1 \leq x \leq \frac{3}{2}$
$\therefore D_f = \left[1, \frac{3}{2}\right]$

77) d Equation of circle is given by:
$(x + 4)(x - 12) + (y - 3)(y + 1) = 0$
$x^2 + y^2 - 8x - 2y - 51 = 0$
Length of y-intercept $= 2\sqrt{f^2 - c} = 2\sqrt{1 + 51} = 2\sqrt{52} = 4\sqrt{13}$

78) d Here, $e = \sqrt{2}$
Distance between directrices $= \frac{2a}{e} = 10$
$2a = 10\sqrt{2}$
Distance between foci $= 2ae = 10\sqrt{2} \times \sqrt{2} = 20$

79) c Dc's of the line making equal angles with the axes is:
$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$
Projection $= \frac{1}{\sqrt{3}}[(a - 1) + (1 + 2) + (0 - 3)]$
$\frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}(a - 1 + 3 - 3)$
$2 = a - 1$
$a = 3$

80) c When an alkene is subjected to acid catalyzed hydration, the intermediate formed is carbocation.
Reactivity of alkene $\propto \frac{1}{\text{Stability of carbocation}} \propto \frac{1}{\text{Inductve effect}}$
Order of inductive effect for carbocation : $3^\circ > 2^\circ > 1^\circ$.
So, Reactivity of alkene follows the order: $1^\circ > 2^\circ > 3^\circ$ i.e., III > II > I.

81) a

82) a $2\ NaHCO_3 \rightarrow Na_2CO_3 + H_2O + CO_2$
2 moles $NaHCO_3$ = 1 mole Na_2CO_3
0.2 mole $NaHCO_3 = \frac{1}{2} \times 0.2 = 0.1$ mole Na_2CO_3

83) a

84) c Molarity of the given Na_2CO_3 solution $= \frac{2.65}{106} \times \frac{1}{250} \times 1000 = 0.1$ M
$M_1V_1 = M_2V_2$
$10 \times 0.1 = 1000 \times M_2$
$M_2 = 0.001$ M

85) c Equivalent weight of Cu deposited = No. of faraday passed = 0.5
Weight of Cu = 0.5 × Equivalent weight $= 0.5 \times \frac{63.5}{2} = 0.5 \times 31.75 = 15.875$ g

86) c

87) b If T be the Tension upward in vertical part of string, then for 6 kg block,
$mg - T = 0$
$6 \times 10 - T = 0$
$T = 60\ N$
Tension in horizontal part of string is also 60 N (System is in dynamic equilibrium)
Now for 10 kg block,
$T - f = 0$ (f = frictional force)
$60 - f = 0$
$f = 60\ N$
Hence, $\mu = \frac{f}{R} = \frac{60}{10g} = \frac{60}{100} = 0.6$

88) c $I'\omega' = I\omega$
$\left(\frac{MR^2}{2} + 2mR^2\right)\omega' = \frac{MR^2}{2}\omega$
$\left(\frac{M+4m}{2}\right)R^2\omega' = \frac{MR^2}{2}\omega$
$\therefore \omega' = \left(\frac{M}{M+4m}\right)\omega$

89) b $v_e = \sqrt{\frac{GM}{R}} \propto \sqrt{\frac{M}{R}}$
$v_e' = \sqrt{\frac{M_m R_e}{M_e R_m}} \times v_e = \frac{2}{9} \times 11 = 2.44\ km/s$

90) a For the particle executing SHM,
$v = \omega\sqrt{A^2 - x^2}$
$\frac{v^2}{\omega^2} = A^2 - x^2$
For $x = x_1, v = v_1$
$\frac{{v_1}^2}{\omega^2} = A^2 - {x_1}^2$ ------ (1)
For $x = x_2, v = v_2$
$\frac{{v_2}^2}{\omega^2} = A^2 - {x_2}^2$ ------ (2)
Subtracting (2) from (1),
$\frac{1}{\omega^2}({v_1}^2 - {v_2}^2) = {x_2}^2 - {x_1}^2$
$\omega^2 = \frac{{v_1}^2 - {v_2}^2}{{x_2}^2 - {x_1}^2}$
$\therefore \omega = \sqrt{\frac{{v_1}^2 - {v_2}^2}{{x_2}^2 - {x_1}^2}}$

91) c mass of steam × latent heat of vaporization = mass of ice × latent heat of fusion + $y \times s \times \Delta\theta$
$x \times 540 = y \times 80 + y \times 1 \times 100$
$\frac{x}{y} = \frac{180}{540}$
$\therefore \frac{x}{y} = \frac{1}{3}$

92) d Rate of heat flow, $R = \frac{Q}{t} = K\,A\frac{\Delta\theta}{l}$
$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)\left(\frac{l_2}{l_1}\right) = \left(\frac{r_1}{r_2}\right)^2\left(\frac{l_2}{l_1}\right) = \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right) = \frac{1}{8}$

93) a $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
$\frac{1}{\left(\frac{2}{3}R\right)} = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$
$\frac{3}{2R} = (\mu - 1).\frac{2}{R}$
$\frac{3}{4} = \mu - 1$
$\mu = \frac{7}{4} = 1.75$

94) b For the position of maxima,
$y_n = \frac{nD}{d}\lambda$
$x = \frac{3 \times 2}{0.2 \times 10^{-3}} \times 5 \times 10^{-7} = 1.5 \times 10^{-3}\ m = 1.5\ cm$

95) c $n' = \frac{v}{v - v_s}n = \frac{300}{300 - 200} \times 400 = 1200$ Hz

96) c For third charge q to be in equilibrium,
$\frac{1}{4\pi\varepsilon_0}\left[\frac{9qe}{x^2} - \frac{qe}{(a-x)^2}\right] = 0$
$\left(\frac{a-x}{x}\right)^2 = \frac{1}{9}$
$\frac{a-x}{x} = \frac{1}{3}$
$\therefore x = \frac{3}{4}a$

97) c $R = 3 + \frac{10 \times (3+R)}{10+3+R} = 3 + \frac{30+10R}{13+R} = \frac{69+13R}{13+R}$
$13R + R^2 = 69 + 13R$
$R^2 = 69$
$\therefore R = \sqrt{69}\ \Omega$

98) d $B = B_1 - B_2 = \frac{\mu_0 N}{2}\left(\frac{I_1}{r_1} - \frac{I_2}{r_2}\right) = \frac{10\mu_0}{2}\left(\frac{0.2}{0.2} - \frac{0.3}{0.4}\right) = 5\mu_0\left(1 - \frac{3}{4}\right) = 5\mu_0\left(\frac{1}{4}\right) = \frac{5}{4}\mu_0$

99) a $I = \frac{V}{Z} = \frac{260}{\sqrt{50^2+120^2}} = \frac{260}{130} = 2$ A

100) a Here, $n = \frac{72000}{24000} = 3$

$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

Thank You!!!!!!