INSTITUTE OF ENGINEERING

Model Entrance Exam

(Set-10 Solutions)

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Date: 2080/04/20
(August-05)

Duration: 2 hours
Time: 8 AM – 10 AM

Section-A (1 marks)

1) c
2) a If the expression 'must' is used in the statement, we use 'need' forms in the tag.
3) b If the verb is in gerund form, we use a possessive adjective and not a possessive pronoun.
4) a
5) a
6) a
7) d
8) a
9) d
10) a
11) c
12) b
13) c $\lim_{x\to 0}\frac{\sin px}{\tan 3x}=4$

$\lim_{x\to 0}\frac{\frac{\sin px}{px}.px}{\frac{\tan 3x}{3x}.3x}=4$

$\frac{p}{3}=4$

$p=12$

14) a $y=\log\sqrt{\tan x}$

$y=\log(\tan x)^{1/2}$

$y=\frac{1}{2}\log(\tan x)$

$\frac{dy}{dx}=\frac{1}{2}\cdot\frac{1}{\tan x}\cdot sec^2x$

At $x=\frac{\pi}{4}$,

$\frac{dy}{dx}=\frac{1}{2}\cdot\frac{1}{1}\cdot(\sqrt{2})^2=1$

15) a If the expression $z=ax+by$ be such that the product of x and y is 1, then the minimum value of $z=2\sqrt{ab}$.

Here, $tan^2\theta.cot^2\theta=1$

∴ Minimum value $=2\sqrt{9\times 4}=12$

16) b $\int e^x(\cos x-\sin x)dx=\int e^x[\cos x+(-\sin x)]dx=e^x\cos x+c$

$\because \int e^x[f(x)+f'(x)]dx=e^xf(x)+c$

17) b $\int_{\pi/6}^{\pi/2}\frac{\cos x}{sin^2x}dx=\int_{\pi/6}^{\pi/2}\text{cosec}\,x\,.\cot x\,dx=|-\text{cosec}\,x|_{\pi/6}^{\pi/2}=-\text{cosec}\frac{\pi}{2}+\text{cosec}\frac{\pi}{6}=-1+2=1$

18) d $x^2-p(x+1)-q=0$

$x^2-px-p-q=0$

$\alpha+\beta=p$ and $\alpha\beta=-p-q$

$(\alpha+1)(\beta+1)=\alpha\beta+\alpha+\beta+1=-p-q+p+1=1-q$

19) b $x=1+\frac{2}{1!}+\frac{4}{2!}+\frac{8}{3!}+\cdots=e^2$

$\sqrt{x}=\sqrt{e^2}=e$

20) b From end, $a=86, d=-4$

$T_{19}=a+18d=86+18(-4)=86-72=14$

21) b $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{-3}=\cos(-\pi)+i\sin(-\pi)$

$\because (\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$ [De Moivre's Theroem]

$=-1+0=-1$

22) b $A^2=A.A=(AB).A=A.(BA)=A.B=A$

23) a There are 6 letters in the word 'GARDEN' which can be arranged in $6!=720$ ways.

There are two vowels 'A' and 'E'. In half of these arrangements A is always before E.

Hence, total number of required arrangements is $\frac{1}{2}(720)=360$.

24) d Since, maximum and minimum values of $\cos x-\sin x$ are $\sqrt{2}$ and $-\sqrt{2}$ respectively.

So, the range of $f(x)$ is $[\sqrt{2}, -\sqrt{2}]$.

25) b Coeff. of x, y, z = dr's of normal to the plane = $(4-1, 13-2, 5-0) = (3, 11, 5)$

26) b $\theta = \tan^{-1} 3$

$\tan\theta = 3$

$\frac{y}{x} = 3$

$y = 3x$

27) d In x-axis, $y = 0$

$x^2 = 4(0+9)$

$x = 6$

When, $x = 6, 6 + ky = 6$

$ky = 0$, which is true $\forall\ k \in R$ as $y = 0$.

28) c $xy = 0 \Rightarrow x = 0\ or\ y = 0$

It represents yz and zx plane i.e., the two planes are at right angles.

29) d $x^2 - y^2 = 0$ does not represent a hyperbola.

It represents a pair of straight lines.

i.e., $(x+y)(x-y) = 0$

30) d Given vectors are parallel if:

$\frac{2}{4} = \frac{1}{-\lambda} = \frac{3}{6}$

$\lambda = -2$

31) b $4\sin^{-1} x + \cos^{-1} x = \pi$

$3\sin^{-1} x + (\sin^{-1} x + \cos^{-1} x) = \pi$

$3\sin^{-1} x + \frac{\pi}{2} = \pi$

$\sin^{-1} x = \frac{\pi}{6}$

$x = \sin\frac{\pi}{6} = \frac{1}{2}$

32) d $\cos\theta = x + \frac{1}{x} = \frac{x^2+1}{x}$

$x^2 - \cos\theta . x + 1 = 0$

Since, x is real;

$B^2 - 4AC \geq 0$

$cos^2\theta - 4 \geq 0$

$cos^2\theta \geq 4$, which is not possible for any value of θ.

33) d $x + 4 + x - 6 = 0$

$2x = 2$

$x = +1$

34) b Electronic configuration of Cl:

$1s^2 2s^2 2p^6 3s^2 3p^5$

$l = 0$ for s-subshell.

No. of electrons $= 2 + 2 + 2 = 6$

35) a As s-character in hybrid orbital increases, the bond angle increases. Since, the s-orbitals overlap from end to end in most situations, it leads to an increase in bond angle.

36) c The presence of calcium and magnesium bicarbonates $Ca(HCO_3)_2$ and $Mg(HCO_3)_2$ causes temporary hardness in water. The presence of soluble salts of calcium and magnesium, i.e., sulphates and chlorides of calcium and magnesium cause permanent hardness in water.

37) c When nitrate salts are treated with conc. H_2SO_4, it gives brown fumes of NO_2.

$2\ KNO_3 + H_2SO_4 \rightarrow 2\ KHSO_4 + HNO_3$

$4\ HNO_3 \rightarrow 4\ NO_2 + O_2 + 2\ H_2O$

Brown fumes

38) a $CaCN_2 + C$: Nitrolime

$CuFeS_2 + FeS_2$: Fool's gold

39) d As we go down the group, the solubility of group IIA hydroxides increases.

40) b In a carbanion, carbon is bonded with three atoms or groups (trivalent) and has eight electrons (octet). It is an electron rich species. It is sp^3 hybridized and has pyramidal geometry.

41) a Alcohol and Ether are functional isomers.

42) c $CuSO_4 \rightarrow Cu^{2+} + SO_4^{2-}$
Valency = 2 (bivalent)

43) c Electronegativity increases from left to right along a period and decreases on descending a group.

44) c Frame-I
- Observer on the train
- Observer and object have same horizontal velocity (velocity of train). So, the object seems to be horizontally no moving. But the object gains vertical velocity.
- So, the observer sees the object falling vertically in a straight line.

Frame-II
- Observer on platform
- Observer is at rest but object has both horizontal velocity (velocity of train) and vertical velocity.
- So, the observer sees the object moving in a parabolic path.

45) a K.E. = $\frac{1}{2}\frac{P^2}{m} = \frac{1}{2} \times \frac{500^2}{10} = 1.25 \times 10^3$ ergs

46) c $g_{poles} > g_{equator}$
- due to smaller polar radius
- due to no effect of earth's rotation on poles.
$g_{poles} - g_{equator} = 0.52 \text{ m/s}^2$

47) d Surface tension = $\frac{Force}{Length\ in\ contact}$
Force = Surface tension × Length in contact
= Surface tension × Circumference of plate
= 75 × 2π × 5 = 750π dynes

48) d E = F/q = N/C, E = V/d = Volt/meter

49) c

50) b Angle of dip is the angle made by resultant magnetic field with horizontal.
$\tan\theta = \frac{V}{H} = 1$
$\therefore \theta = 45^\circ$
At poles, $\theta = 90^\circ$ and at equator $\theta = 0^\circ$

51) b All potentials (electric, electrostatic) are scalar and all gradients (temperature, velocity) are vector.

52) a

Mirror	Nature of image of real object
Convex	Virtual, erect, diminished
Concave	a. Real, inverted, magnified b. Virtual, erect and diminished

53) d If thin metal foil is inserted in the middle, then capacitance remains constant.
$C = \frac{\varepsilon_0 A}{d}$
After insertion,
$C_1 = \frac{\varepsilon_0 A}{d/2} = 2C$
$C_2 = \frac{\varepsilon_0 A}{d/2} = 2C$
Since, the two capacitors will be in series, equivalent capacitance. C_s is given by
$C_s = \frac{C_1 C_2}{C_1 + C_2} = C$

54) c An intrinsic semiconductor having trivalent impurity is called P-type semiconductor.

55) c -A perfectly black body emits radiations of all possible wavelengths when it is hot so it appears white.
-A perfectly black body absorbs all incident radiations without any reflection and transmission so it appears black when cools.
-The radiations emitted by perfectly black body depends on its surface temperature and not on nature of material.

56) d -If one slit is illuminated by red colour and another by violet colour, then no interference is observed as the sources are incoherent.
-In interference, energy is redistributed.
-Interference is based on conservation of energy.

57) d -It shows no particular direction at earth magnetic pole as H = 0 at magnetic poles. So, it may stay in any direction.
-At equator, V = 0

58) a -Twinkling of stars is due to refraction. It is due to refractive index fluctuation of atmosphere.
-A stick partially dipped in water seems bent due to refraction.
-Appearance of sun. just before actual sunrise and just after sunset is due to refraction
-A tank of liquid or a pond appears shallow than its actual depth due to refraction.
-Sun appears to be elliptical when its at the horizon due to refraction.

59) c

60) c In β-emission, a neutron of nucleus decays into a proton, a β-particle and an anti-neutrino.

$n \rightarrow p + e^- + \bar{\nu}$

<u>Section-B</u> (2 marks)

61) d

62) a

63) d

64) d The enthusiastic tone of the passage seems meant to encourage people to adopt retired greyhounds. Choice a is wrong because there is only one statistic in the passage (in the first sentence), and it is not used to prove the point that greyhounds make good pets. Choice b is wrong because the author substantiates every point with information. Choice c is wrong because the passage does make the negative point that greyhounds do not make good watchdogs.

65) d Given, $2a = 6, 2b = 4\ i.e., a = 3, b = 2$

$e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{4}{9} = \frac{5}{9}$

$e = \frac{\sqrt{5}}{3}$

Distance between the pins $= 2ae = 2 \times 3 \times \frac{\sqrt{5}}{3} = 2\sqrt{5}$

66) b As given.

$\sqrt{\lambda^2 - c} = \sqrt{(-\mu)^2 + c}$

$\lambda^2 - c = \mu^2 + c$

$\lambda^2 - \mu^2 = 2c$

∴ Locus of (λ, μ) is: $x^2 - y^2 = 2c$

67) c $a^2x^2 + 2h(a+b)xy + b^2y^2 = 0$ --- (i)

$ax^2 + 2hxy + by^2 = 0$ --- (ii)

Equation of bisectors of angle between the line pair (i) is:

$h(a+b)(x^2 - y^2) = (a^2 - b^2)xy$

$h(x^2 - y^2) = (a - b)xy$

which is same as the equation of bisectors between the line pair (ii).

68) a Midpoint of the line joining the given points lie on the line $y = 2x + c$

$\therefore \frac{3+b}{2} = 2\left(\frac{a+5}{2}\right) + c$

$2a + 2c - b + 7 = 0$ --- (i)

Also, given line passes through (a, b)

$\therefore b = 2a + c$ --- (ii)

Solving (i) and (ii),

$c = -7$

69) a $\sin\left(\frac{\pi}{4}\cot\theta\right) = \cos\left(\frac{\pi}{4}\tan\theta\right)$

$\sin\left(\frac{\pi}{4}\cot\theta\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{4}\tan\theta\right)$

$\frac{\pi}{4}\cot\theta = \frac{\pi}{2} - \frac{\pi}{4}\tan\theta$

$\cot\theta + \tan\theta = 2$

$\frac{1}{\tan\theta} + \tan\theta = 2$

$tan^2\theta - 2\tan\theta + 1 = 0$

$tan^2\theta - \tan\theta - \tan\theta + 1 = 0$

$\tan\theta(\tan\theta - 1) - 1(\tan\theta - 1) = 0$

$(\tan\theta - 1)(\tan\theta - 1) = 0$

$\tan\theta = 1$

$\tan\theta = \tan\frac{\pi}{4}$

$\theta = n\pi + \frac{\pi}{4}$

70) b $\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-c)}$

$\frac{\sin(B+C)}{\sin(A+B)} = \frac{\sin(A-B)}{\sin(B-c)}$ $\because A+B+C=\pi$

$\sin(B+C).\sin(B-C) = \sin(A+B).\sin(A-B)$

$sin^2B - sin^2C = sin^2A - sin^2B$

$b^2 - c^2 = a^2 - b^2$

$a^2 + c^2 = 2b^2$

i.e., a^2, b^2, c^2 are in A.P.

71) b As given,

$(\vec{a} - 4\vec{b}).(7\vec{a} - 2\vec{b}) = 0$

$7a^2 - 2\,\vec{a}.\vec{b} - 28\,\vec{a}.\vec{b} + 8b^2 = 0$

$7a^2 + 8b^2 = 30\,\vec{a}.\vec{b}$

$7.1 + 8.1 = 30(1.1.\cos\theta)$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

72) b Area $= \int_0^{\pi/2} sin^2x\,dx = \frac{1}{2}\int_0^{\pi/2}(1-\cos 2x)\,dx = \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi/2} = \frac{1}{2}\left[\left(\frac{\pi}{2} - 0\right) - \frac{1}{2}(0-0)\right] = \frac{\pi}{4}$

73) c $\int \frac{\cos 2x - \cos 2y}{\cos x - \cos y}dx = = \int \frac{(2cos^2x - 1) - (2cos^2y - 1)}{\cos x - \cos y}dx = \int \frac{2cos^2x - 2cos^2y}{\cos x - \cos y}dx = 2\int(\cos x + \cos y)\,dx = 2(\sin x + x\cos y) + c$

74) b Slope of the line $y = x$ is 1.

As given, slope of tangent is perpendicular to this line i.e.,

$\frac{dy}{dx} = -1$

Diff. given curve, we get,

$\frac{dy}{dx} = 2x - 3$

$-1 = 2x - 3$

$x = 1$

Putting $x = 1$ in the equation of curve, $y = 0$

Hence, required point is (1, 0).

75) b Here, $\frac{x^2}{y} = \frac{4a^2t^2}{2at^2}$

$\frac{x^2}{y} = 2a$

$y = \frac{x^2}{2a}$

$\frac{dy}{dx} = \frac{2x}{2a} = \frac{x}{a}$

76) d $\lim_{x\to 0}\frac{2^x - 1}{(1+x)^{1/2} - 1}$ (0/0 form)

$= \lim_{x\to 0}\frac{2^x \log 2}{\frac{1}{2}(1+x)^{-1/2}}$ (Applying L'Hospital rule)

$= \frac{2^0 \log 2}{\frac{1}{2}(1+0)^{-1/2}} = \frac{\log 2}{\frac{1}{2}} = 2\log 2 = \log 2^2 = \log 4$

77) c Since, co-domain $= \left[0, \frac{\pi}{2}\right)$, for f be onto,

Range $= \left[0, \frac{\pi}{2}\right)$

This is possible only when $x^2 + x + a \geq 0\ \forall\ x \in R$

Thus, $1^2 - 4a \leq 0$ (discriminant ≤ 0)

$a \geq \frac{1}{4}$

78) a If x occurs in T_{r+1}, $r = \frac{5(2)-1}{2+1} = 3$

∴ Coefficient of x = $^5C_3\,(k)^3$ = 270

$k^3 = 27$

$k = 3$

79) b We know that,

$A\,(adj\,A) = |A|I$

Thus, $\lambda = |A| = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = cos^2x + sin^2x = 1$

80) a

81) c

82) d 18 g H_2O contains 2 g H

0.72 g H_2O contain $\frac{2}{18} \times 0.72 = 0.08$ g H

44 g CO_2 contains 12 g C

3.08 g CO_2 contains $\frac{12}{44} \times 3.08 = 0.84$ g C

$C: H = \frac{0.84}{12} : \frac{0.08}{1} = 0.07 : 0.08 = 7 : 8$

∴ Empirical formula $= C_7H_8$

83) b **Key concept:**

Hybridization = no. of σ-bond + lone pair

Hybridization = 2 (sp)

Hybridization = 3 (sp^2)

Hybridization = 4 (sp^3)

In $NO_3{}^-$, there are 3 bonding domains in the central N atom (one single bond and two double bonds) and zero lone electron pairs. (sp)

In $NO_2{}^-$, there are 2 bonding domains in the central N atom (one single bond and one double bond) and zero lone electron pairs. (sp^2)

In $NH_4{}^-$, there are 4 bonding domains in the central N atom (four single bonds) and zero lone electron pairs. (sp^3)

84) a $N_1V_1 = N_2V_2$

$N_1 = \frac{N_2V_2}{V_1} = \frac{0.4\times20}{40} = 0.2$

Hence, $M = \frac{0.2}{2} = 0.1$

85) b $p^H = 12$

$[H^+] = 10^{-12}\ M$

$[OH^-] = 10^{-2}\ M$

$Ba(OH)_2 \rightleftharpoons Ba^{2+} + 2OH^-$

$[Ba^{2+}] = \frac{10^{-2}}{2} = 5 \times 10^{-3}$ M

$K_{sp} = (5 \times 10^{-3})(10^{-2})^2 = 5 \times 10^{-7}$

86) b

87) c $v_0 = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$ for first line of Balmer series.

For doubly ionized Li^{++},

$v = Z^2R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = (3)^2R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

Now, $\frac{v}{v_0} = 9 \Rightarrow v = 9v_0$

88) b $n = \frac{t}{t_{1/2}} = \frac{3240}{1620} = 2$

As, $\frac{N}{N_0} = \frac{m}{m_0} = \left(\frac{1}{2}\right)^n$

Mass of radium left after 2 half-lives is:

a) $m = m_0\left(\frac{1}{2}\right)^n = 1 \times \left(\frac{1}{2}\right)^2 = \frac{1}{4} = 0.25$ mg

89) c Capacitance of a parallel plate capacitor is:

$C = \frac{\varepsilon_0 A}{d}$ --- (i)

Potential difference between the plates is:

$V = Ed$ --- (ii)

Energy stored in the capacitor is:

$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\varepsilon_0 E^2 Ad$

90) c Total resistance = 2.5 + 0.5 = 3 kΩ = 3000 Ω

Current (I) = 6/3000 A

Reading of voltmeter = $I \times (2.5 \times 1000) = \frac{6}{3000} \times 2500 = 5$ V

91) a Here, $l = 50\ cm = 0.5\ m, M = 10^6\ Am^{-1}$

As, $M = \frac{\text{Magnetization current } (I_m)}{\text{length } (l)}$

$I_M = M \times l = 10^6 \times 0.5 = 5 \times 10^5$ A

92) b $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

$T_2 = \frac{Q_2}{Q_1} \times T_1 = \frac{500}{750} \times 410 = 273.33\ K = 273.33 - 273 = 0.33\ ^\circ C$

93) d $Q = mL = 12 \times 80 = 960\ cal$

Also, $Q = \frac{KA(T_1 - T_2)t}{x}$

$960 = \frac{K \times 5 \times 10^{-4} \times 100 \times 60}{0.25}$

$K = \frac{960 \times 0.25}{5 \times 10^{-2} \times 60} = 80\ cals^{-1}m^{-1}{}^\circ C^{-1}$

94) b $v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2 \times 10^{11}}{8000}} = \frac{1}{2} \times 10^4$ m/s

$\therefore t = \frac{x}{v} = \frac{1}{\frac{1}{2} \times 10^4} = 2 \times 10^{-4}$ s

95) a Fringe width (β) = $\frac{\lambda D}{d}$

or, $\lambda = \frac{\beta d}{D} = \frac{11780 \times 10^{-10} \times 10^{-4}}{2 \times 10^{-4}} = 5890 \times 10^{-10} = 5890$ Å

96) b As the image formed is real, therefore lens must be convex, $v = 20\ cm$. Let f_1 be the focal length for this lens.

$\frac{1}{f_1} = \frac{1}{v} - \frac{1}{u} = \frac{1}{20} - \frac{1}{u}$

After placing it in contact with another lens, the image shifted to 10 cm towards the combination.

i.e., $v = (20 - 10) = 10\ cm$

So, $\frac{1}{10} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$

$\frac{1}{10} - \frac{1}{u} = \left(\frac{1}{20} - \frac{1}{u}\right) + \frac{1}{f_2}$

$f_2 = 20\ cm = \frac{20}{100}\ m$

$\therefore P = \frac{1}{f_2} = \frac{100}{20} = 5\ D$

97) c In a stationary lift, $T = 2\pi\sqrt{\frac{l}{g}}$

In upward moving lift, $T' = 2\pi\sqrt{\frac{l}{g+a}}$

$\frac{T'}{T} = \sqrt{\frac{g}{g+a}} = \sqrt{\frac{g}{g+\frac{g}{4}}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$

$\therefore T' = \frac{2T}{\sqrt{5}}$

98) c Resultant downward force along the incline = $mg(sin\theta - \mu cos\theta)$

Normal reaction = $mg\ cos\theta$

Given, $mg\ cos\theta = 2mg(sin\theta - \mu cos\theta)$

or, $mg \cos\theta + 2\mu mg\ cos\theta = 2mg\ sin\theta$

or, $mg \cos\theta + 2 \times \frac{1}{2} \times mg\ cos\theta = 2mg\ sin\theta$

or, $mg \cos\theta + mg\ cos\theta = 2mg\ sin\theta$

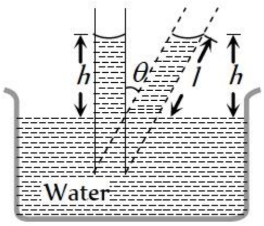
or, $2mg \cos\theta = 2mg\ sin\theta$

or, $\frac{sin\theta}{cos\theta} = \frac{2mg}{2mg}$

or, $tan\theta = 1$

$\therefore \theta = 45^\circ$

99) b



Vertical height of the water in the tube remains constant.

So, $l = \frac{h}{c0s\theta} = \frac{3}{cos60^{\circ}} = \frac{3}{\frac{1}{2}} = 3 \times 2 = 6$ cm

100) d Let the positive direction of motion be from south to north.

Velocity of train A w.r.t. ground $(v_{AG}) = +54\ km/hr = +54 \times \frac{5}{18} m/s = +15\ m/s$

Velocity of train B w.r.t. ground $(v_{BG}) = -90 \frac{km}{hr} = -90 \times \frac{5}{18} m/s = -25\ m/s$

Relative velocity of train A with respect to train B is:

$v_{AB} = v_{AG} + v_{GB} = v_{AG} - v_{BG} = 15 - (-25) = 40\ m/s$

Thank You!!!!!!