BEATS ENGINEERING

INSTITUTE OF ENGINEERING

Model Entrance Exam

(Set-12 Solutions)

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Date: 2080/05/02
(August-19)

Duration: 2 hours
Time: 8 AM – 10 AM

Section-A (1 marks)

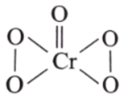
1) d IF_3 → sp^{3}d hybridization (2 lone pair + 3 bond pair) → bent-T geometry
PCl_3 → sp^3 hybridization (1 lone pair + 3 bond pair) → pyramidal geometry
NH_3 → sp^3 hybridization (1 lone pair + 3 bond pair) → pyramidal geometry
BF_3 → sp^2 hybridization (0 lone pair + 3 bond pair) → trigonal planar geometry

2) b Number of atoms = number of moles × N_A × atomicity $= 0.1 \times 6.023 \times 10^{23} \times 3 = 1.806 \times 10^{23}$ atoms

3) c The highest pH refers to the basic solution containing OH^- ions. Therefore, the basic salt releasing more OH^- ions on hydrolysis will give highest pH in water.
Only the salt of strong base and weak acid would release more OH^- ion on hydrolysis. Among the given salts, Na_2CO_3 corresponds to the basic salt as it is formed by the neutralization of NaOH [strong base] and H_2CO_3 [weak acid].
$CO_3{}^{2-} + H_2O \rightleftharpoons HCO_3{}^- + OH^-$

4) c The electronic configuration $1s^2, 2s^2 2p^5, 3s^1$ shows lowest ionization energy because this configuration is unstable due to the presence of one electron in s-orbital. Hence, less energy is required to remove the electron.

5) c The structure of CrO_5 is:



Oxidation state of Cr is +6 due to the presence of two peroxide linkages which can be calculated as:
$x + 4(-1) + (-2) = 0$
$x - 6 = 0$
$x = +6$

6) c $Be(OH)_2$ is amphoteric in nature as it reacts with both acid and alkali.
$Be(OH)_2 + HCl \rightarrow BeCl_2 + 2H_2O$
$Be(OH)_2 + 2NaOH \rightarrow Na_2[Be(OH)_4]$
This amphoteric nature of Be is due to small size of Be. The other hydroxides of alkaline earth metals are basic in nature

7) b CH_3CHO and $C_6H_5CH_2CHO$ both being aliphatic aldehydes react with Tollen's reagent, Fehling solution and Benedict test. So, these reagents cannot be used to distinguish them. CH_3CHO due to the presence of CH_3CO group reacts with NaOH and I_2 to give yellow crystals of Iodoform while $C_6H_5CH_2CHO$ does not react with it.

8) a I effect of $F > H$. So, permanent displacement of σ-electron occurs away from the carbon chain and is more in trifluoroacetic acid.
Strength of acid ∝ -I effect

9) b

10) c Each of the Na^+ and Cl^- ions has coordination number of 6.

11) d Hydrometallurgy involves both leaching and precipitation of the metal from its solution by adding a precipitating agent.

12) d White phosphorous (most reactive phosphorous) produce phosphorescence.

13) b

14) c Mercury in presence of ozone is oxidized to suboxide. It starts sticking to glass and loses mobility. Hence, mercury loses its meniscus in contact with ozone. This is termed as the tailing of mercury and is used as a test for ozone.
$2Hg + O_3 \rightarrow Hg_2O + O_2$

15) b

16) a

17) d

18) b

19) d

20) c

21) d

22) c
23) b
24) a
25) a
26) b
27) c For asinx + bcosx,
Maximum value = $\sqrt{a^2+b^2}$, Minimum value = $-\sqrt{a^2+b^2}$
$\therefore$ sinx + cosx = $\sqrt{1^2+1^2}=\sqrt{2}$
28) b Given, $\sin^{-1}x=\frac{\pi}{5}$
$\Rightarrow \sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$
$\Rightarrow \cos^{-1}x=\frac{\pi}{2}-\frac{\pi}{5}=\frac{3\pi}{10}$
29) c $\cos^2x=\frac{1}{4}$
or, $\cos^2x=\left(\frac{1}{2}\right)^2$
or, $\cos^2x=\cos^2\left(\frac{\pi}{3}\right)$
$\therefore x=n\pi\pm\frac{\pi}{3}$
30) b $\vec{b}=-4\vec{a}$
$\therefore\ \vec{a}\,||\,\vec{b}$
31) a $\lim_{x\to0}\frac{\sin ax}{\sin bx}=\frac{a}{b}$
$\therefore \lim_{x\to0}\frac{\sin 7x}{\sin 5x}=\frac{7}{5}$
32) d $\frac{dy}{dz}=\frac{\frac{dy}{dx}}{\frac{dz}{dx}}=\frac{\frac{d\tan^{-1}x}{dx}}{\frac{d\cot^{-1}x}{dx}}=\frac{\frac{1}{1+x^2}}{-\frac{1}{1+x^2}}=-1$
33) b $\int\frac{1}{x\log x}dx=\int\frac{\frac{1}{x}}{\log x}dx=\log\log x+c$
34) d $y=f(x)=x^3+3x^2-9x+25$
$f'(x)=3x^2+6x-9$
$f''(x)=6x+6$
For point of inflection. $f''(x)=0$
$\Rightarrow 6x+6=0$
$\Rightarrow x=-1$
35) c Given, ax+4y=5
$\Rightarrow \frac{x}{5/a}+\frac{y}{5/4}=1$
Since, X-intercept = 3
$\Rightarrow \frac{5}{a}=3$
$\therefore a=\frac{5}{3}$
36) d Two lines are coincident if
$h^2=ab$
$\Rightarrow \left(\frac{-k}{2}\right)^2=(1).(4)$
$\Rightarrow k^2=16$
$\Rightarrow k=4$
37) d Given equation of circle is:
$x^2+y^2-2\lambda x-2\lambda y+\lambda^2=0$
Comparing with, $x^2+y^2+2gx+2fy+c=0$
g = λ, f = λ, c = λ^2
Here, $g^2=f^2=c$
$\therefore$ Circle touches both the axes.
38) d Eccentricity of the parabola is always 1.
39) c $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=(1-\cos^2\alpha)+(1-\cos^2\beta)+(1-\cos^2\gamma)$
$=3-(\cos^2\alpha+\cos^2\beta+\cos^2\gamma)=3-1=2$
40) a $\sigma=\sqrt{\frac{250}{10}}=5$
Coefficient of variance $=\frac{\sigma}{\bar{X}}\times100=\frac{5}{50}\times100=10$
41) d $A-(B\cap C)=\{x:x\in A \text{ and } x\notin(B\cap C)\}$

$= \{x: x \in A \text{ and } x \notin B \text{ or } x \in A \text{ and } x \notin C\}$
$= (A - B) \cup (A - C)$

42) b f(x) is defined for all values except x = 1
So, domain of f = R – {1}

43) b $\frac{1+2i}{1-i} = \frac{1+2i}{1-i} \times \frac{1+i}{1+i} = \frac{1+i+2i+2i^2}{1-i^2} = \frac{1+3i+2(-1)}{1-(-1)} = \frac{-1+3i}{2} = -\frac{1}{2} + \frac{3}{2}i$
$\left(-\frac{1}{2}, \frac{3}{2}\right)$ lies in second quadrant.

44) a Let three numbers in G.P. be $\frac{a}{r}, a, ar$.
Product = 1728
i.e., $\frac{a}{r}.a.ar = 1728$
or, $a^3 = 1728$
$\therefore a = 12$ (middle term)

45) c Here, all elements below the leading diagonal are zero.
Hence, it is an upper triangular matrix.

46) b We know, $\alpha + \beta = -\frac{b}{a}$
or, $\alpha - \alpha = -\frac{b}{a}$
or, $0 = -\frac{b}{a}$
$\therefore b = 0$

47) c Quark combination of proton = uud
Quark combination of neutron = udd
Quark combination of antineutron = $\bar{u}\bar{d}\bar{d}$
Baryon : formed by 3 quarks
The baryon number of each quark = $\frac{1}{3}$
Meson : Formed by one quark and one anti-quark

48) a The quantity of electricity is charge.
q = it
[q] = $[M^0L^0TA]$

49) b

50) b

51) c Due to low density, clouds have very small terminal velocity so they fall slowly and appear to be floating.

52) b With change of temperature, volume and density changes in reverse direction but mass (i.e., product of volume and density) remains unchanged. So, 50 g (given mass) weighs equal in summer and in winter.

53) b $C = \sqrt{\frac{3PV}{M}} \propto \sqrt{P}$ (since M and V be constant)
So, $\frac{C}{C_0} = \sqrt{\frac{4}{1}}$
$\Rightarrow C = 2C_0$

54) b $\lambda_{medium} = \frac{\lambda_{vacuum}}{\mu} = \frac{6000}{2}\text{Å} = 3000\ \text{Å}$

55) a

56) b $f = \frac{v}{4L} = \frac{340}{4 \times 0.25} = 340$ Hz

57) d

58) a $F = qvB \sin 0° = 0$

59) c For wattless circuit, phase difference between current and voltage should be π/2. Hence, resistance R should be zero as $\cos\phi = \frac{R}{Z} = 0$.

60) b

Section-B (2 marks)

61) a The passage is organized chronologically. The steps for starting a book club are listed in the order in which they should occur.

62) c The second sentence of the second paragraph states this clearly.

63) d Deciding on the club's focus—the kinds of books or genre the club will read—should be done prior to this meeting and prior to recruiting members, according to the second paragraph.

64) d The tone and specificity of the passage infer that a successful book club requires careful planning. The tone and specificity of the passage infer that a successful book club requires careful planning.

65) d For the reaction,

$Ca(OH)_2(s) \rightleftharpoons Ca^{2+}(aq) + 2OH^-(aq)$

S 2S

$K_{sp} = [Ca^{2+}][OH^-]^2 = S(2S)^2$ --- (i)

Given, $p^H = 9$

$p^{OH} = 14 - -9 = 5$

$[OH^-] = 10^{-5}$

From (i), $[OH^-] = 2S = 10^{-5}$

$S = \frac{10^{-5}}{2}$

$K_{sp} = 4S^3 = 4\left(\frac{10^{-5}}{2}\right) = 0.5 \times 10^{-15}$

66) b $E^0{}_X = -1.2\ V$

$E^0{}_Y = +0.5\ V$

$E^0{}_Z = -3.0\ V$

Higher the reduction potential, lesser is the reducing power.

$\therefore Z > X > Y$

67) a $\frac{r_{CH_4}}{r_X} = 2 = \sqrt{\frac{M_X}{M_{CH_4}}} = \sqrt{\frac{M_X}{16}}$

$\frac{M_X}{16} = 4$

$M_X = 64$

68) b $k = \frac{1}{40} \ln \frac{0.1}{0.025} = \frac{1}{40} \ln 4$

$R = k[A]^1 = \frac{1}{40} \ln 4 \times 0.01 = 3.47 \times 10^{-4}$ M/min

69) c

70) c

71) c

72) a $C_6H_5CHO + HCHO \xrightarrow{aq.NaOH} C_6H_5CH_2OH + HCOONa$

(Cannizzaro reaction)

73) c $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= \left(\frac{-b}{a}\right)^3 - \frac{3c}{a}\left(\frac{-b}{a}\right) = \frac{3abc - b^3}{a^3}$

74) c $\frac{(1+x+x^2)}{e^x} = (1 + x + x^2).e^{-x}$

$= (1 + x + x^2).\left\{1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\right\}$

Coefficient of $x^2 = 1.\frac{1}{2!} + 1\left(\frac{-1}{1!}\right) + 1.1 = \frac{1}{2}$

75) b Here, $P(Q) = \frac{4}{52}$

Since, one card is already picked,

$p(J) = \frac{4}{51}$

$\therefore P(Q\ and\ J) = \frac{4}{52} \times \frac{4}{51} = \frac{4}{663}$

76) b $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$

Applying $C_1 \rightarrow C_1 - (C_2 + C_3)$, we get,

$= \begin{vmatrix} -2a & c+a & a+b \\ -2p & r+p & p+q \\ -2x & z+x & x+y \end{vmatrix}$

$= -2\begin{vmatrix} a & c+a & a+b \\ p & r+p & p+q \\ x & z+x & x+y \end{vmatrix}$

Operate $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_2$

$= -2\begin{vmatrix} a & c & b \\ p & r & q \\ x & z & y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

77) b

Players (14)	Defenders (5)	Non-defenders (9)	Selection
	4	7	$^5C_4 \times {}^9C_7 = 180$
	5	6	$^5C5 \times {}^9C_6 = 84$

Total number of ways = 180 + 84 = 264

78) b $\tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)=4$

$\Rightarrow \frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}.\tan\theta}+\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}.\tan\theta}=4$

$\Rightarrow \frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta}=4$

$\Rightarrow \frac{(1+\tan\theta)^2+(1-\tan\theta)^2}{1-\tan^2\theta}=4$

$\Rightarrow \frac{1+2\tan\theta+\tan^2\theta+1-2\tan\theta+\tan^2\theta}{1-\tan^2\theta}=4$

$\Rightarrow 2+2\tan^2\theta=4-4\tan^2\theta$

$\Rightarrow 6\tan^2\theta=2$

$\Rightarrow \tan^2\theta=\frac{2}{6}=\frac{1}{3}$

$\Rightarrow \theta=n\pi\pm\frac{\pi}{6}$

79) d Let m_1 and m_2 be the slopes of the lines represented by $4x^2+2hxy-7y^2=0$

$m_1+m_2=-\frac{2h}{b}=\frac{2h}{7}$

$m_1m_2=\frac{a}{b}=-\frac{4}{7}$

As given, $\frac{2h}{7}=-\frac{4}{7}$

$\Rightarrow h=-2$

80) d Solving the line $y=x+a\sqrt{2}$ and the circle $x^2+y^2=a^2$, we get,

$x^2+(x+a\sqrt{2})^2=a^2$

$\Rightarrow x^2+x^2+2ax\sqrt{2}+2a^2=a^2$

$\Rightarrow 2x^2+2ax\sqrt{2}+a^2=0$

$\Rightarrow \left(\sqrt{2}x\right)^2+2\left(\sqrt{2}x\right)a+a^2=0$

$\Rightarrow \left(\sqrt{2}x+a\right)^2=0$

$\Rightarrow x=-\frac{a}{\sqrt{2}}$

$\therefore y=-\frac{a}{\sqrt{2}}+a\sqrt{2}=\frac{a}{\sqrt{2}}$

Hence, point of contact $=\left(-\frac{a}{\sqrt{2}},\frac{a}{\sqrt{2}}\right)$

81) b As given, $2b=8\Rightarrow b=4$ and $e=\frac{\sqrt{5}}{3}$

$\because e^2=1-\frac{b^2}{a^2}$

$\Rightarrow \frac{5}{9}=1-\frac{16}{a^2}$

$\Rightarrow a=6$

$\therefore$ major axis $=2a=2\times6=12$

82) a The line $y=4x$ meets $y=x^3$ at $4x=x^3$.

$\therefore x=0,2,-2\Rightarrow y=0,8,-8$

$\Rightarrow A=\int_0^2(4x-x^3)=\left(2x^2-\frac{x^4}{4}\right)_0^2=4$ sq. units

83) d $f(x)+f(1-x)-2=0$

or, $f(x)-1+f(1-x)-1=0$

or, $g(x)+g(1-x)=0$

Replacing x by $x+\frac{1}{2}$, we get,

$g\left(\frac{1}{2}+x\right)+g\left(\frac{1}{2}-x\right)=0$

So, it is symmetrical about $\left(\frac{1}{2},0\right)$.

84) b $\lim_{x\to1}\frac{1-x^2}{\sin2\pi x}=-\lim_{x\to1}\frac{2\pi\ (1-x)(1+x)}{2\pi\ \sin(2\pi-2\pi x)}\lim_{x\to1}\frac{(2\pi-2\pi x)}{\sin(2\pi-2\pi x)}\frac{1+x}{2\pi}=-\frac{1}{\pi}$

85) a $(\sin x)(\cos y)=1/2$

$\frac{dy}{dx}[(\sin x)(\cos y)]=\frac{dy}{dx}\left(\frac{1}{2}\right)$

or, $\cos x.\cos y\frac{dy}{dx}-\sin y.\sin x\frac{dy}{dx}=0$

or, $\frac{dy}{dx} = (\cot x)(\cot y)$
or, $\frac{d^2y}{dx^2} = -\text{coesc}^2 x.\cot y\frac{dy}{dx} - \text{cosec}^2 y.\cot x\frac{dy}{dx}$
Now, $\left(\frac{dy}{dx}\right)_{(\pi/4,\pi/4)} = \cot\left(\frac{\pi}{4}\right).\cot\left(\frac{\pi}{4}\right) = 1.1 = 1$
$\left(\frac{d^2y}{dx^2}\right)_{(\pi/4,\pi/4)} = -(2).(1).(1) - (2).(1).(1) = -2 - 2 = -4$

86) c $f(x) = \frac{t+3x-x^2}{x-4}$
$f'(x) = \frac{(x-4)(3-2x)-(t+3x-x^2)}{(x-4)^2}$
For maximum or minimum, $f'(x) = 0$
or, $-2x^2 + 11x - 12 - t - 3x + x^2 = 0$
or, $-x^2 + 8x - (12 + t) = 0$
For one maxima and minima, $D > 0$
or, $64 - 4(12 + t) > 0$
or, $16 - 12 - t > 0$
or, $4 > t$
or, $t < 4$
Hence, range of t is: $(-\infty, 4)$

87) c $\int \frac{e^x}{\sqrt{4-e^{2x}}}dx = \int \frac{e^x}{\sqrt{2^2-(e^x)^2}}dx$
Let $e^x = t$
or, $e^x dx = dt$
$\therefore I = \int \frac{dt}{\sqrt{4-t^2}}dx = \int \frac{dt}{\sqrt{2^2-(t)^2}}dx = \sin^{-1}\left(\frac{t}{2}\right) + c = \sin^{-1}\left(\frac{e^x}{2}\right) + c$

88) a $r = 100\ cm = 1\ m$
Frequency, $f = \frac{14}{22}$ rps
$\omega = 2\pi f = 2 \times \frac{22}{7} \times \frac{14}{22} = 4\ rads^{-1}$
The acceleration of the stone is: $a_c = \omega^2 r = (4)^2 \times 1 = 16\ m/s^2$

89) b Here, $m = 0.2$ kg, $v = 5\ m/s$, h = length of elevator = 5 m
As relative velocity of the bolt w.r.t. elevator is zero, therefore, in the impact, only potential energy of the bolt is converted into heat energy.
Amount of heat produced = Potential energy lost by the bolt = $mgh = 0.2 \times 10 \times 5 = 10$ J

90) c As Young's modulus, $Y = \frac{(F/A)}{(\Delta L/L)}$
As applied force and extension ΔL are same for steel and copper wires,
$\frac{F}{\Delta L} = \frac{YA}{L}$
So, $\frac{Y_S A_S}{L_S} = \frac{Y_C A_C}{L_C}$
or, $\frac{Y_S}{Y_L} = \frac{L_S}{L_C} \times \frac{A_C}{A_S} = \frac{4.5}{3.5} \times \frac{4\times10^{-5}}{3\times10^{-5}} = 1.7$

91) b Volume of bubble, $V = \frac{4}{3}\pi R^3$
$R = \left(\frac{3V}{4\pi}\right)^{1/3}$
Work done, $W = S \times 8\pi R^2 = S \times 8\pi\left(\frac{3V}{4\pi}\right)^{2/3}$
$W \propto V^{2/3}$
$\therefore \frac{W_2}{W_1} = \left(\frac{V_2}{V_1}\right)^{2/3} = \left(\frac{2V}{V}\right)^{2/3} 2^{2/3}$
$W_2 = 2^{2/3} W_1$

92) d If m is the mass of ice melted, then
Heat spent in melting = Heat supplied by the ball
$mL = Ms\Delta T$
$m \times 80 = (80 \times 1000) \times 0.2 \times 100$
$m = 2 \times 10^4$ g

93) d The coefficient of performance of a refrigerator is given by:
$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1-Q_2}$
$\frac{1}{3} = \frac{Q_2}{200-Q_2}$
$Q_2 = \frac{200}{4} = 50$ J

$\therefore W = Q_1 - Q_2 = 200 - 50 = 150$ J

94) d The given transverse harmonic wave equation is:

$y = 3\sin\left(36t + 0.018x + \frac{\pi}{4}\right)$ --- (i)

As there is positive sign between t and x terms, therefore the given wave is travelling in the negative x-direction.

The standard transverse harmonic wave equation is:

$y = a\sin(\omega t + kx + \phi)$ --- (ii)

Comparing (i) and (ii), we get,

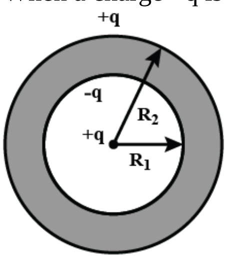
$a = 3\ cm, \omega = 36\ rad\ s^{-1}, k = 0.018\ rad\ cm^{-1}$

Amplitude of the wave, $a = 3\ cm$

Frequency of the wave, $f = \frac{\omega}{2\pi} = \frac{36}{2\pi} = \frac{18}{\pi}$ Hz

Velocity of the wave, $v = \frac{\omega}{k} = \frac{36}{0.018} = 2000\ cms^{-1} = 20\ ms^{-1}$

95) b When a charge +q is placed at the centre of spherical cavity as shown in the figure,



Charge induced on the inner surface of shell $= -q$

Charge induced on the outer surface of shell $= +q$

$\therefore$ Surface charge density on the inner surface $= \frac{-q}{4\pi {R_1}^2}$

Note:

Surface charge density on the outer surface $= \frac{+q}{4\pi {R_2}^2}$

96) a Applying Kirchhoff's first law at the junction P,

$6 = i_1 + i_2$ --- (1)

Applying Kirchhoff's second law to the closed loop PQRP,

$-2i_1 - 2i_1 + 2i_2 = 0$

$4i_1 - 2i_2 = 0$ --- (2)

Solving (1) and (2), we get

$i_1 = 2A, i_2 = 4A$

97) c For six layers of windings, total number of turns, $N = 6 \times 450 = 2700$

Number of turns per unit length, $n = \frac{N}{l} = \frac{2700}{90\times10^{-2}} = 3000$

The field inside the solenoid near the centre, $B = \mu_0 nI = 4\pi \times 10^{-7} \times 3000 \times 6 = 72\pi \times 10^{-4}$ T

98) b $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Here, $f = 20\ cm, \mu = 1.55, R_1 = R$ and $R_2 = -R$

$\frac{1}{20} = (1.55 - 1)\left(\frac{1}{R} - \frac{1}{(-R)}\right)$

$\frac{1}{20} = 0.55 \times \frac{2}{R}$

$\therefore R = 0.55 \times 2 \times 20 = 22\ cm$

99) b Distance of 2nd order maximum from the centre of the screen

$x = \frac{5}{2}\frac{D\lambda}{d}$

$d = \frac{5}{2}\frac{D\lambda}{x} = \frac{5}{2} \times \frac{0.8\times600\times10^{-9}}{15\times10^{-3}} = 80\ \mu m$

100) a Current gain, $\alpha = \frac{Power\ gain}{Voltage\ gain} = \frac{800}{840} = \frac{20}{21}$

$\beta = \frac{\alpha}{1-\alpha} = \frac{\frac{20}{21}}{1-\frac{20}{21}} = 20$

As, $\beta = \frac{I_C}{I_B}$

$I_C = \beta I_B = 20 \times 1.2 = 24\ mA$

Thank You!!!!!!