INSTITUTE OF ENGINEERING

Model Entrance Exam

(Set-13 Solutions)

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Date: 2080/05/09
(August-26)

Duration: 2 hours
Time: 8 AM – 10 AM

Section-A (1 marks)

1) c
2) d
3) d
4) a
5) a
6) d
7) a
8) c
9) b
10) a
11) b
12) c
13) c From triangle of vector,
$\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=0$
$\left|\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}\right|^2=0$
$(AB)^2+(BC)^2+(CA)^2+2\left(\overrightarrow{AB}.\overrightarrow{BC}+\overrightarrow{BC}.\overrightarrow{CA}+\overrightarrow{CA}.\overrightarrow{AB}\right)=0$
$a^2+a^2+a^2+2\left(\overrightarrow{AB}.\overrightarrow{BC}+\overrightarrow{BC}.\overrightarrow{CA}+\overrightarrow{CA}.\overrightarrow{AB}\right)=0$
$\overrightarrow{AB}.\overrightarrow{BC}+\overrightarrow{BC}.\overrightarrow{CA}+\overrightarrow{CA}.\overrightarrow{AB}=\frac{-3a^2}{2}$
14) c $x^2+y^2=0$
$\Rightarrow x=0$ & $y=0$, which represents z-axis.
15) c For both conics, centre = (0,0) and vertices = $(\pm 5,0)$. 'e' is different in both cases. So, foci and directrices are different.
16) c As, $2x+y+\lambda=0$
$y=-2x-\lambda$
$\therefore m=-2, a=-2, c=\lambda$
For normal, $c=2am-am^3$
$-\lambda=-2(-2)(-2)+2(-2)^3$
$\lambda=24$
17) b Put $x=r\cos\theta, y=r\sin\theta$
$r^2=x^2+y^2$
$x^2+y^2+4x-6y-2=0$
Centre = (-2, 3)
18) b $y=mx+c$
Two parameters m and c should be known.
19) b Here, $n(S)=49$
Favorable numbers are $11, 21, 31, 41$
$\therefore$ Required probability $=\frac{4}{49}$
20) a On squaring, we get,
$1+2\sin\theta.\cos\theta=1+2\sin 2\theta.\cos 2\theta$
$\sin 2\theta=2\sin 2\theta.\cos 2\theta$
$\cos 2\theta=\frac{1}{2}$
$\cos 2\theta=\cos\frac{\pi}{3}$
$2\theta=\frac{\pi}{3}$
$\theta=\frac{\pi}{6}$
21) c $\cos^{-1}\frac{1}{x}+\cos^{-1}\frac{1}{y}=\sec^{-1}x+\sec^{-1}y=\text{cosec}^{-1}y+\sec^{-1}y=\frac{\pi}{2}$
22) d $\lim_{x\to 0}\frac{e^x-e^{-x}}{\sin x}$ $\left[\frac{0}{0}\text{ form}\right]$

$= \lim_{x\to 0} \frac{e^x - e^{-x}}{\cos x} = \frac{e^0 + e^{-0}}{\cos 0} = \frac{1+1}{1} = 2$

23) b At $x = 2$, $f(x)$ is undefined, $f(x)$ is discontinuous at $x = 2$.

24) a $y = e^{\sqrt{2x}}$

$\frac{dy}{dx} = \frac{d\left(e^{\sqrt{2x}}\right)}{d(\sqrt{2x})} \cdot \frac{d\left(\sqrt{2x}\right)}{d(2x)} \cdot \frac{d(2x)}{d(x)} = e^{\sqrt{2x}} . \frac{1}{2}(2x)^{-\frac{1}{2}} . 2 = \frac{e^{\sqrt{2x}}}{\sqrt{2x}}$

25) c $\int \frac{1+cos^2x}{sin^2x} dx = \int (cosec^2x + cot^2x)\, dx = \int (cosec^2x + cosec^2x - 1)\, dx = \int (2cosec^2x - 1)\, dx = -2\cot x - x + c$

26) d $\int_0^a \frac{dx}{a^2+x^2} = \left[\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]_0^a = \frac{1}{a}\tan^{-1}\left(\frac{a}{a}\right) = \frac{1}{a}\tan^{-1} 1 = \frac{1}{a} . \frac{\pi}{4} = \frac{\pi}{4a}$

27) d $y = x^3 + 3x^2 - 9x + 2$

$y' = 3x^2 + 6x - 9$

$y'' = 6x + 6$

At point of inflection,

$y'' = 0$

$6x + 6 = 0$

$x = -6$

28) d Let A = {1} which is non-empty set with least number of elements. Then, its subsets are the set itself {1} and empty set { }. So, the least number of subsets of a non-empty set is 2.

29) a $f(x) = e^x + 1$

Range of e^x is $(0, \infty)$

So, Range of $(e^x + 1)$ is $(0 + 1, \infty + 1)$ i.e., $(1, \infty)$.

30) c $|A - \lambda I| = 0$

$\begin{vmatrix} -3-\lambda & 4 \\ 2 & -1-\lambda \end{vmatrix} = 0$

$\lambda = 1, -5$

31) d $x + iy = a - ib$

Taking conjugate on both sides,

$x - iy = a + ib$

32) b $x^2 - (p + 1)x + pq = 0$

Sum of the roots $=$ p $+ 1$

i.e., p $+$ q $=$ p $+ 1$

q $= 1$

33) a

34) d

35) b

36) b Difference between apparent and real depth of a pond is due to refraction. Other three are due to total internal reflection.

37) d When an ac voltage of 220 V is applied to a capacitor C, the charge on the plates is in phase with the applied voltage. As the circuit is pure capacitive so, the current developed leads the applied voltage by a phase angle of 90˚. Hence, power delivered to the capacitor per cycle is:

$P = V_{rms} I_{rms} \cos 90^\circ = 0$

38) b An emf is induced only when magnetic flux linked with the loop changes. This is possible when the loop is rotated about a diameter.

39) c

40) b When the string vibrates in loops n, its frequency is:

$f_n = \frac{nv}{2L}$

Where, L is the length of the string and v is the velocity of the wave.

∴ When the string fixed at its both ends vibrate in 1 loop, 2 loops, 3 loops and 4 loops, the frequencies are in the ratio $1:2:3:4$

41) a When the lift is at rest, the frequency of oscillation of simple pendulum is, $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$.

When the lift falls freely, then effective value of acceleration due to gravity is: $g' = g - a = g - g = 0$.

$\therefore f = \frac{1}{2\pi}\sqrt{\frac{0}{L}} = 0$

42) a $\eta = 1 - \frac{T_2}{T_1} = \frac{T_1 - T_2}{T_1}$

When T_1 and T_2 both are decreased by 100 K each, $(T_1 - T_2)$ stays constant. T_1 decreases.

$\therefore \eta$ increases.

43) b As, $\beta = 2\alpha, \gamma = 3\alpha$;

$\frac{\beta}{\gamma} = \frac{2\alpha}{3\alpha} = \frac{2}{3}$

44) c When a body is just floating in a liquid whose density is equal to the density of body, is pushed down slightly, then downward force on the body increases due to atmospheric pressure and due to water column above the body. As a result of which, the body sinks in the liquid.

45) d Linear momentum is not conserved.

46) a As, $x \propto t^3$, $v \propto 3t^2$, $a \propto 6t$ (increasing with time)

47) b When water freezes to form ice, the randomness decreases, i.e., entropy decreases.

48) d In face-centered cubic lattice, the structure possesses 8 corner atoms and 6 atoms at the centre of each face. Thus, the total number of metal atoms per unit cell in face-centered cubic lattice $= 8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$

49) c 106 g Na_2CO_3 contains $3 \times 6.023 \times 10^{23}$ oxygen atoms.

10.6 g Na_2CO_3 contains $\frac{3 \times 6.023 \times 10^{23}}{106} \times 10.6$ oxygen atoms $= 1.806 \times 10^{23}$ oxygen atoms

50) d Maximum number of electrons in a sub-shell $= 2(2l + 1) = 2(2.3 + 1) = 14$

51) c Bond angle decreases with decrease in electronegativity of the central atom.

52) c Hydrogen cannot be prepared by the action of dil. H_2SO_4 on copper and mercury as these two metals cannot displace hydrogen from acids. Action of dil. H_2SO_4 on Pb stops after sometime due to the formation of insoluble $PbSO_4$. Only iron reacts rapidly with dil. H_2SO_4 to give H_2.

53) a

54) c At cathode,

$2\ H_2O + 2e^- \rightarrow H_2 + 2OH^-$

$Na^+ + OH^- \rightarrow NaOH$

At anode,

$Br^- \rightarrow Br + e^-$

$Br + Br \rightarrow Br_2$

55) b

56) b

57) d If an organic compound contains S, in addition to N, then NaCNS is formed which reacts with $FeCl_3$ to form red coloured ferric sulphocyanide.

$Na + C + N + S \rightarrow NaCNS$

$3NaCNS + FeCl_3 \rightarrow Fe(CNS)_3 + 3NaCl$

58) b

59) b All aldehydes (aliphatic and aromatic) and ketones (aliphatic methyl ketones) gives sodium bisulphite test.

60) c Order of basicity:

$2^\circ > 1^\circ > 3^\circ > NH_3$

<u>Section-B</u> (2 marks)

61) a The examples in this passage are mainly about Roosevelt's accomplishments.

62) c The second sentence of the first paragraph supports this choice.

63) b In the second paragraph, the first sentence supports this answer.

64) a This is the only choice and is stated in paragraph 1.

65) a Acute angle between the line $x^2 + 4yx + y^2 = 0$ is

$\tan^{-1}\left(\frac{2\sqrt{4-1}}{1+1}\right) = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

Angle bisector of $x^2 + 4yx + y^2$ are:

$2(x^2 - y^2) = (1 - 1)xy$

$x^2 = y^2$

$x = \pm y$

As, $x + y = 0$ is perpendicular to $x - y = 4$, the given triangle is isosceles with vertical angle equal to $\pi/3$ and hence equilateral.

66) b Centre $= (0, 10)$

Radius $= \sqrt{100 - 90} = \sqrt{10}$

Since the line lies outside the circle,

$p > a$

$\left|\frac{0-10}{\sqrt{1+m^2}}\right| > \sqrt{10}$

$\frac{10}{\sqrt{1+m^2}} > \sqrt{10}$

$100 > 10 + 10m^2$

$90 > 10m^2$

$9 > m^2$

$|m| < 3$

67) b Vertex $= (0, 0)$

$4a = 12 \Rightarrow a = 3$

Ends of latus rectum $= (\pm 2a, a) = (6, 3)$ & $(-6, 3)$

$\therefore$ Required area $= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 6 & 3 & 1 \\ -6 & 3 & 1 \end{vmatrix} = 18$ sq. units

68) d Here, $l = \frac{-3}{\sqrt{(-3)^2+(2)^2+(6)^2}} = \frac{-3}{7}$

$m = \frac{2}{\sqrt{(-3)^2+(2)^2+(6)^2}} = \frac{2}{7}$

$n = \frac{6}{\sqrt{(-3)^2+(2)^2+(6)^2}} = \frac{6}{7}$

$p = 7$

Hence, equation of plane in normal form will be:

$-\frac{3}{2}x + \frac{2}{7}x + \frac{6}{7}z = 7$

$-3x + 2y + 6z - 49 = 0$

69) c $\vec{a}$ lies in the plane of $\vec{b}$ and $\vec{c} \Rightarrow \vec{a}, \vec{b}, \vec{c}$ are coplanar.

$[\vec{a}\ \vec{b}\ \vec{c}] = 0$

$\Rightarrow \vec{a}.(\vec{b} \times \vec{c}) = 0$

70) a $\bar{X} = \frac{\sum x_i}{n}$

$\sum x_i = 36 \times 50 = 1800$

Now, $\sum x_i - (42 + 30) = 1800 - 72 = 1728$

$\therefore$ New mean $= \frac{1728}{48} = 36$

71) a $2(r + R) = 2\left[(s - c)\tan\frac{C}{2} + \frac{c}{2\sin C}\right]$

$= 2\left[(s - c) + \frac{c}{2}\right]$ $\left(\because C = \frac{\pi}{2}\right)$

$= 2s - 2c + c$

$= a + b + c - c$ $(\because 2s = a + b + c)$

$= a + b$

72) d $\frac{C^{20}_{r-1}}{C^{20}_{r}} = \frac{1}{2}$

$\frac{20!}{(r-1)!(21-r)!} \times \frac{r!(20-r)!}{20!}$

$\frac{1}{2} = \frac{r}{21-r}$

$r = 7$

73) b Given, $T_n = \frac{1}{2}\left[\frac{3^n}{n!} - \frac{1}{n!}\right]$

Sum $= \sum_{n=1}^{\infty} T_n = \frac{1}{2}\left[\sum_{n=1}^{\infty}\frac{3^n}{n!} - \sum_{n=1}^{\infty}\frac{1}{n!}\right]$

$= \frac{1}{2}\left[\left\{1+\frac{3}{1!}+\frac{3^2}{2!}+\cdots\right\}\left\{1+\frac{1}{1!}+\frac{1}{2!}+\cdots\right\}\right]$

$= \frac{1}{2}(e^3-e)$

74) a

Ladies (4)	Gentlemen(6)	Number of Selection
2	3	${}^4C_2 \times {}^6C_3 = 120$
3	2	${}^4C_3 \times {}^6C_2 = 60$
4	1	${}^4C_4 \times {}^6C_1 = 6$

Total $= 120+60+6 = 186$

75) d $fog(x) = gof(x)$

$f(2^x) = g(x^2)$

$(2^x)^2 = 2^{x^2}$

Applying log on both sides,

$2x\log 2 = x^2 \log 2$

$2x - x^2 = 0$

$x(2-x) = 0$

$x = 0, 2$

Solution set $= \{0, 2\}$

76) c $3+3\alpha+3\alpha^2+\cdots\infty = \frac{45}{8}$

$3(1+\alpha+\alpha^2+\cdots\infty) = \frac{45}{8}$

$3\left(\frac{1}{1-\alpha}\right) = \frac{45}{8}$

$\frac{24}{45} = 1-\alpha$

$\alpha = \frac{7}{15}$

77) d $\lim_{x\to 0}\left(\frac{1+2x}{1-3x}\right)^{\frac{1}{x}} = \frac{\lim_{x\to 0}\left[(1+2x)^{\frac{1}{2x}}\right]^2}{\lim_{x\to 0}\left[(1-3x)^{-\frac{1}{3x}}\right]^{-3}} = \frac{e^2}{e^{-3}} = e^5$

78) c $x = a(1+\sin t)$

$\frac{dx}{dt} = a + a\cos t$

$y = a(1-\cos t)$

$\frac{dy}{dt} = 0 + a\sin t$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\sin t}{a(1+\cos t)} = \frac{a.2\sin\frac{t}{2}\cos\frac{t}{2}}{a.2cos^2\frac{t}{2}} = \tan\frac{t}{2}$

79) b Area $= \int_0^b y\,dx = \int_0^b \frac{x^2}{4b}dx = \frac{1}{4b}\left[\frac{x^3}{b}\right]_0^b = \frac{1}{4b}.\frac{b^3}{3} = \frac{b^2}{12}$

80) a $\frac{dy}{dx} + P(x)y = Q(x)$

I. F. $= e^{P(x)dx}$

Given:

$\frac{dy}{dx}(xlog\ x) + y = 2\log x$

$\frac{dy}{dx} + \frac{y}{x\log x} = \frac{2}{x}$

P(x) $= \frac{1}{x\log x}$

I. F. $= e^{\frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$

81) b As the number of undecayed nuclei decreases from 25% to 12.5% in 10 s, it shows that the half-life of the sample is 10 s.

i.e., $T_{1/2} = 10\ s$

Decay constant, $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{10}$

Mean life, $\tau = \frac{1}{\lambda} = \frac{10}{0.693} = 14.43\ s$

82) b $\frac{\lambda}{\lambda_0} = \frac{\left[\frac{1}{2^2}-\frac{1}{3^2}\right]}{\left[\frac{1}{2^2}-\frac{1}{4^2}\right]} = \frac{5}{36} \times \frac{16}{3} = \frac{20}{27}$

$\lambda = (20/27)\lambda_0$

83) b As reflected light is completely polarized, therefore, $i_p = 60^\circ$

$\mu = \tan i_p = \tan 60^\circ = \sqrt{3}$

As, $\mu = \frac{c}{v}$

$v = \frac{c}{\mu} = \frac{3\times10^8}{\sqrt{3}} = \sqrt{3} \times 10^8$ m/s

84) a For minimum deviation, $i = e$

$r_1 = r_2 = \frac{A}{2} = \frac{60^\circ}{2} = 30^\circ$

Using Snell's law at the interface,

$1 \sin i = \sqrt{3} \sin r_1$

$\sin i = \sqrt{3} \sin 30^\circ = \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$

$i = 60^\circ$

85) d Here, $n = 25$ turns, $r = 12$ cm, $B = 0.5$ T

Since the coil is placed in a uniform magnetic field normal to the plane of the coil, the angle between magnetic moment and magnetic field direction is zero i.e., $\theta = 0^\circ$

$\therefore \tau = MB \sin\theta = MB \sin 0^\circ = 0$

86) a $I = \frac{E}{R+r}$

For the maximum current from the battery,

$R = 0$

i.e., $I = \frac{E}{r} = \frac{24}{0.8} = 30$ A

87) d For a thin uniformly charged spherical shell, the field points outside the shell at a distance x from the centre is:

$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}$

If the radius of the sphere is R, $Q = \sigma 4\pi R^2$

$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 4\pi R^2}{x^2} = \frac{\sigma R^2}{\varepsilon_0 x^2}$

This is inversely proportional to square of the distance from the centre. It is as if the whole charge is concentrated at the centre.

88) c Here, Frequency of source, $f = 400$ Hz

Speed of sound, $v = 330$ m/s

Speed of source (i.e., train), $v_s = 30$ m/s

As the source is moving towards stationary observer

$f' = \frac{v}{v-v_s} \times f = \frac{330}{330-30} \times 400 = 440$ Hz

89) d According to an ideal gas equation,

$PV = nRT$

$V = \frac{nRT}{P}$

$V = \frac{nrT^2}{P}$ $\quad \because P = \frac{a}{T}$ (Given)

$dV = \frac{2nRT}{a} dT$

Work done by the gas, $dW = P\,dV$

$W = \int_T^{4T} \frac{a}{T}\frac{2nRT}{a} dT = [2nRT]_T^{4T} = 8nRT - 2nRT = 6nRT$

90) c Breaking force = Breaking stress × Area of cross section

For a given material of the wire, breaking stress is constant.

$\therefore \frac{F_1}{A_1} = \frac{F_2}{A_2}$

$F_2 = F_1\left(\frac{A_1/4}{A_1}\right) = \frac{F_1}{4} = \frac{W}{4}$ $\quad [\because A = \pi r^2]$

91) c Acceleration of the solid sphere when it rolls without slipping down an inclined plane is

$a = \frac{g \sin\theta}{1+\frac{I}{MR^2}}$

For a solid sphere, $I = \frac{2}{5}MR^2$

$a = \frac{g \sin\theta}{1+\frac{2}{5}} = \frac{5}{7}\ g \sin\theta$

Acceleration of the same sphere when it slides without friction down an same inclined plane is

$a' = g \sin\theta$

So, $\frac{a'}{a} = \frac{\frac{5}{7} g \sin\theta}{g \sin\theta} = \frac{7}{5}$

$a' = \frac{7}{5}a$

92) d Since ranges are same so shell have been fired at complementary angles. Let, they be θ and $90^\circ - \theta$.

$t_1 = \frac{2u \sin\theta}{g}$ and $t_2 = \frac{2u \sin(90^\circ-\theta)}{g} = \frac{2u \cos\theta}{g}$

$t_1 t_2 = \frac{2}{g} . \frac{2u^2 \sin\theta \cos\theta}{g} = \frac{2}{g} . \frac{u^2 \sin 2\theta}{g} = \frac{2R}{g}$ $\left[\because R = \frac{u^2 \sin 2\theta}{g}\right]$

93) a $\mathrm{Mg} + \frac{1}{2}\mathrm{O_2} \rightarrow \mathrm{MgO}$

24 g 16 g

24 g Mg react with 16 g of O_2

1 g Mg reacts with $\frac{16}{24} = \frac{2}{3} = 0.67$ g of O_2

However, only 0.56 g of O_2 is present, thus Mg is present in excess.

0.67 g of O_2 reacts with 1 g of Mg

0.56 g of O_2 with $\frac{1}{0.67} \times 0.56 = \frac{56}{67} = 0.84$ g Mg

Mg left $= 1 - 0.84 = 0.16$ g

94) b $2MnO_4{}^- + Br^- + H_2O \rightarrow 2MnO_2 + BrO_3{}^- + 2OH^-$

+7 +4

95) b $K_c = \frac{[NO_2]^2}{[N_2O_4]} = \frac{(1.2\times10^{-2})^2}{4.8\times10^{-2}} = 3 \times 10^{-3}\ mol\ L^{-1}$

96) a $W = ZIt$

$It = \frac{W}{Z} = 1$ $\because (W = Z)$

$t = \frac{1}{I} = \frac{1}{0.25} = 4$ A

97) b $3Mg + N_2 \rightarrow Mg_3N_2$

$Mg_3N_2 + H_2O \rightarrow Mg(OH)_2 + NH_3$

$CuSO_4 + 4NH_3 \rightarrow Cu[(NH_3)_4]SO_4$

Deep blue colour

98) a Sulphur trioxide is prepared by heating conc. H_2SO_4 with large excess of phosphorous pentoxide, P_2O_5.

$H_2SO_4 + P_2O_5 \rightarrow SO_3 + 2HPO_3$

99) a

100) d $\mathrm{CH_3CH_2OH} \xrightarrow{\mathrm{P+I_2}} CH_3CH_2I \xrightarrow{\mathrm{Mg,ether}} CH_3CH_2MgI \xrightarrow{\mathrm{HCHO}} CH_3CH_2CH_2OMgI \xrightarrow{\mathrm{H_2O}} CH_3CH_2CH_2OH$

Thank You!!!!!!