INSTITUTE OF ENGINEERING

MODEL ENTRANCE EXAM

(SET – 3)
Solutions

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Date : 2081/02/26
(June 08)

Duration : 2 hours
Time : 8 A.M. – 10 A.M.

SECTION – A (1 marks) (1*60 = 60)

1) b
2) c
3) b
4) a
5) b
6) c
7) c
8) d
9) a
10) b
11) a
12) d
13) d The contrapositive of "if p, then q" is: "if ~q, then ~p".
14) c
15) c
16) c $\sin^{-1}\left(\sin\frac{5\pi}{3}\right) = \sin^{-1}\left\{\sin\left(2\pi - \frac{\pi}{3}\right)\right\} = \sin^{-1}\left\{\sin\left(-\frac{\pi}{3}\right)\right\}$
Principal value $= -\frac{\pi}{3}$ which lies in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
17) a Coefficient of x^2 + Coefficient of $y^2 = 0$
i.e., $a + b = 0$
18) b For point P(x, y) with $y = 6$
$y^2 = 12x$
$6^2 = 12x$
$x = 3$
$\therefore P = (3, 6)$
Also, focus = (3, 0)
∴ Focal distance = 6
19) d $\lim_{x\to\infty} x\tan\frac{1}{x} = \lim_{\frac{1}{x}\to 0}\frac{\tan\frac{1}{x}}{\frac{1}{x}} = 1$
20) b $y = \sin^{-1}(\cos x)$
$y = \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - x\right)\right\} = \left(\frac{\pi}{2} - x\right)$
$\therefore \frac{dy}{dx} = -1$
21) d $\int(1 + x + x^2 + x^3 + \cdots + \infty)dx = \int\frac{1}{1-x}dx = -log_e(1-x) + c = log_e(1-x)^{-1} + c$
$= log_e(1 + x + x^2 + x^3 + \cdots\infty) + c$
22) a $|\vec{a}| = |\vec{b}| = k$ (suppose)
Also, $\vec{a}.\vec{b} = 0$
Now, $\vec{a}.(\vec{a} + \vec{b}) = |\vec{a}||\vec{a} + \vec{b}|\cos\theta$
$\vec{a}.\vec{a} + \vec{a}.\vec{b} = |\vec{a}||\vec{a} + \vec{b}|\cos\theta$
$|\vec{a}|^2 = |\vec{a}||\vec{a} + \vec{b}|\cos\theta$
$\cos\theta = \frac{k}{|\vec{a}+\vec{b}|} = \frac{k}{\sqrt{k^2+k^2}}$ ($\because$ $\vec{a}$ and $\vec{b}$ are perpendicular)
$= \frac{1}{\sqrt{2}}$
$\therefore \theta = \frac{\pi}{4}$
23) d $P(A|B) = \frac{P(A\cap B)}{\boldsymbol{P(B)}} = \frac{0.15}{0.60} = \frac{1}{4}$

24) a $y = e.e^{log_e x} = e.x$

$\therefore \frac{dy}{dx} = e$

25) $\int \frac{1}{\sqrt{x^2-a^2}} dx = \log\left(x + \sqrt{x^2 - a^2}\right) + c$

26) c Distance from y-aixs = $|x|$

27) b $\frac{x^2}{\left(\frac{1}{4}\right)^2} + \frac{y^2}{\left(\frac{1}{5}\right)^2} = 1$

$\frac{1}{4} > \frac{1}{5}$. So, it is a horizontal ellipse.

28) d Coefficient matrix (A) = $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} = 0$

So, A^{-1} does not exist.

29) c 6 is repeated maximum number of times. So, mode is 6.

30) b $\log\left(\frac{p}{q}.\frac{q}{r}.\frac{r}{p}\right) = \log 1 = 0$

31) b $e^{-\log 2} = e^{\log\left(\frac{1}{2}\right)} = \frac{1}{2}$

32) c Minimum value $= \frac{1}{\text{Maximum value}} = \frac{1}{\sqrt{3^2+4^2+5}} = \frac{1}{10}$

33) c Order of velocity : $\gamma > \beta > \alpha$

Particle with high velocity stay for very short time near the atom of medium so possibility of ionization is less energy loss is gradual so they can penetrate more.

$\therefore Penetrating\ power:\ \gamma > \beta > \alpha$

34) b

35) c Lenz's law gives direction of induced current.

36) c

37) d As the electric field is uniform, equal force acts throughout the field i.e., the lines of force are parallel.

38) c Air column vibrates, not the liquid column.

39) b

40) d

41) a $\left(\frac{dQ}{dt}\right) = \sigma AT^4 \propto A$

Sphere has minimum surface area, so rate of cooling is slowest.

42) c

43) a Spring balance now reads the wight of both stone and container.

44) b In parallel combination of cells, the voltage across the terminals is same and resistance is minimum. Therefore, from $V = IR$, the current drawn from cell combination will be more.

45) b

46) c In XY-plane, a vector is $3\hat{\imath} + \hat{\jmath}$.

Length in XY-plane $= \sqrt{3^2 + 1^2} = \sqrt{10}$

47) a

48) b

49) b

50) a

51) a ${NH_4}^+$ is not a Lewis acid. A species acts as a Lewis acid only when it has empty orbital which ${NH_4}^+$ does not have. ${NH_4}^+$ is an Arrhenius acid as well as Bronsted-Lowry acid.

52) b The orbital associated with $n = 3$ $l = 1$ is 3p. The number of electrons with $m = -1$ is 2.

53) d $NaClO_4$ is a salt made from one of the strongest acid $HClO_4$. Hence, in aqueous solution, $NaClO_4$ will have the lowest pH.

54) a Actually in case of a chemical reaction, the more reactive element displaces the less reactive element from its salt solution. In this case, nickel would be able to displace silver from its salt solution i.e. from the silver nitrate form.

55) d Aqueous solution of weak acid and strong base is basic in nature.

56) a Unit of rate constant = $(molL^{-1})^{1-n}sec^{-1}$
For first order reaction,
$n = 1$
Unit of rate constant = sec^{-1}

57) d

58) c Extraction of copper from copper pyrite ($CuFeS_2$) involves : crushing followed by concentration of the ore by froth flotation.

59) b Ammonium nitrite $(NH_4Cl + NaNO_2)$ gives N_2 on heating.

60) a On moving down the group, basic character increases and acidic character decreases.

<u>SECTION – B (2 marks)</u> (2*40=80)

61) d

62) a

63) b

64) d

65) d Let, $y = \frac{1}{1-x} \Rightarrow x = \frac{y}{1+y}$ defined for all but $y \neq -1$.
$\therefore$ Range $= (-\infty, -1) \cup (-1, \infty)$

66) a If 2 men and 2 women are selected at a time, 2 mixed double can be organized.
So, total number of ways $= 2 \times\times C(6,2) \times C(4,2) = 180$

67) a Use options and form a trial.
For, $n = 3$, $1^2 + 2^2 + 3^2 = 14$
Put, $n = 3$, option (a) $= \frac{3\times4\times7}{6} = 14$

68) c Mean $(\bar{X}) = \frac{2+4+6+8+10}{5} = 6$

$\bar{X}$	$(X - \bar{X})^2$
2	16
4	4
6	0
8	4
10	16

Variance $= \frac{\sum(X-\bar{X})^2}{n} = \frac{40}{5} = 8$

69) a $q = 0.5, n = 40, p = 0.5$
Mean of binomial distribution $= np = 40 \times 0.5 = 20$
Standard equation $= \sqrt{npq} = \sqrt{40 \times 0.5 \times 0.5} = 3.16$

70) b $2s = 5k + 6k + 5k = 16k$
$s = 8k$
Now, $r = \frac{\Delta}{s}$
$6 = \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s} = \frac{\sqrt{8k.3k.2k.3k}}{8k}$
$k = 4$

71) b $y = \mathrm{a}\cos(x + b)$
$\frac{dy}{dx} = -\,\mathrm{a}\sin(x + b)$

$\frac{d^2y}{dx^2} = -\operatorname{acos}(x+b) = -y$

$\therefore \frac{d^2y}{dx^2} + y = 0$

72) d Length of intercept by circle with x-axis is:

$2\sqrt{g^2 - c}$

A circle touches x-axis, length of intercept = 0

$g^2 = c$

73) a If $a > b$, foci are $A(ae, 0)$ and $B(-ae, 0)$ and positive end of minor axis is $C(0, b)$.\

Now, $AC \perp BC$

Slope of AC . Slope of BC = -1

$b^2 = a^2e^2$

Also, $b^2 = a^2(1-e^2)$

$a^2e^2 = a^2 - a^2e^2$

$2e^2 = 1$

$e = \frac{1}{\sqrt{2}}$

74) a $PM = \frac{2}{\sqrt{3}}, QN = \frac{2}{\sqrt{3}}$

$PQ = \sqrt{2}, RQ = 0$

Required projection:

$PR = MN = \sqrt{PQ^2 - QR^2} = \sqrt{\left(\sqrt{2}\right)^2 - 0} = \sqrt{2}$

75) b $\lim\limits_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0-\Delta x)}{2\Delta x} = \frac{1}{2}\left[\lim\limits_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} + \lim\limits_{\Delta x \to 0} \frac{f(x_0-\Delta x) - f(x_0)}{-\Delta x}\right]$

$= \frac{1}{2}[f'(x_0) + f'(x_0)] = f'(x_0)$

76) d $I = \int \frac{dx}{(x-1)\sqrt{x^2-1}} = \int \frac{dx}{(x-1)^2\sqrt{\frac{x+1}{x-1}}}$

Put, $u = \frac{x+1}{x-1}$

$du =. \frac{(x-1)1-(x+1)1}{(x-1)^2} dx = \frac{-2\,dx}{(x-1)^2}$

$I = -\frac{1}{2}\int \frac{du}{\sqrt{u}} = -\sqrt{u} + c = -\sqrt{\frac{x+1}{x-1}} + c$

77) a If $\measuredangle C = \frac{\pi}{2}$, then $A + B = \frac{\pi}{2}$

Let, $y = \cos A . \cos B = \cos A . \cos\left(\frac{\pi}{2} - A\right) = \cos A . \sin A$

$y = \frac{1}{2}\sin 2A$ --- (i)

So, $y' = \frac{1}{2}. 2\cos 2A = \cos 2A$

$y'' = -2\sin 2A$

Now, for extreme value, $y' = 0$

$\cos 2A = 0$

$A = \frac{\pi}{4}$

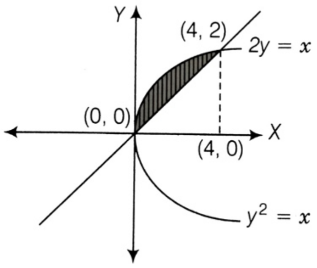
So, $B = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

For, $A = \frac{\pi}{4}, y'' = -2\sin 2\left(\frac{\pi}{4}\right) = -2 < 0$

$\therefore$ for $A = \frac{\pi}{4}$, y is maximum.

$y_{max} = \cos\frac{\pi}{4}.\sin\frac{\pi}{4} = \frac{1}{2}$

78) a



$y^2 = x$ and $2y = x$
We get, $y^2 = 2y$
$y(y-2) = 0$
$y = 0, y = 2$
∴ Curves meet at (0, 0) and (4, 2).
$A = \int_0^4 \left(\sqrt{x} - \frac{x}{2}\right) dx = \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{4}\right]_0^4 = \frac{16}{3} - 4 = \frac{4}{3}$ sq. units

79) c For coplanar vectors
$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$
$a(bc-1) - 1(c-1) + 1(1-b) = 0$
$abc - a - c + 1 + 1 - b = 0$
$a + b + c = abc + 2$

80) c $CH_3CH_2Cl + alc.KOH \rightarrow CH_2 = CH_2 \xrightarrow{O_3/Zn} HCHO + HCHO$

81) b $\mathrm{CH_3COOH} \xrightarrow{\mathrm{NH_3}} CH_3COONH_4 \xrightarrow{\Delta} CH_3CONH_2$

82) a Higher the reduction potential, higher is the tendency to get reduced.
∴ MnO_4^- is the strongest oxidizing agent.

83) a In one molecule of magnesium phosphate, 8 oxygen atoms are present.
Thus, 1 mole of magnesium phosphate will contain 8 moles of oxygen.
Hence, 0.25 moles of oxygen atoms are present in $\frac{1}{8} \times 0.25 = 3.125 \times 10^{-2}$ moles of magnesium phosphate

84) a Since the number of moles of CO and nitrogen are equal, their mole fractions will be 0.5 each. The partial pressure of the nitrogen in the mixture is equal to the product of mole fraction of nitrogen and total pressure.
It is$0.5 \times 1 = 0.5$ atm

85) b 30 mL of 1 N HCl will neutralize 30 mL of 1 N NaOH. Therefore, 20 mL of 1 N HCl will be left unneutralized.
Volume of mixture = 50 + 30 =80 mL
$[HCl] = \frac{20}{80} = 0.25\ M$
$[H^+] = 0.25\ M$
pH $= -\log[H^+] = 0.6021$

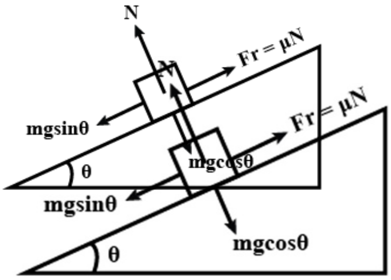
86) a $LiNO_3 \xrightarrow{\Delta} Li_2O + NO_2 + O_2$

87) b $3Cl_2 + 6NaOH\ (aq) \rightarrow 5NaCl + NaClO_3 + 3H_2O$
(Hot) -1 +5

88) c $h = \frac{1}{2}gT^2$
$h' = \frac{1}{2}g\left(\frac{T}{3}\right)^2$

Height from ground $= h - h' = h = \frac{1}{2}gT^2 - \frac{1}{2}g\left(\frac{T}{3}\right)^2 = \frac{1}{2}gT^2\left(1-\frac{1}{9}\right)h = \frac{8}{9}h$

89) c



$mg\sin\theta - f = ma$

$mg\sin\theta - \mu_k mg\cos\theta = ma$

$a = g\sin\theta - \mu_k g\cos\theta = (\sin\theta - \mu_k\cos\theta)g = \left(\frac{\sqrt{3}}{2} - 0.25\times\frac{1}{2}\right)\times 10 = 7.4\ m/s^2$

90) a $v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G\times\frac{4}{3}\pi R^3\rho}{R}} = \sqrt{\frac{8}{3}\ G\pi R^2\rho} = R\sqrt{\frac{8}{3}\ G\pi\rho} \propto R$

$\frac{v_p}{v_e} = \frac{R_p}{R_e} = 2$

$v_p = 2v_e = 22\ km/second$

91) c Weight of sphere = Weight of liquid displaced

$V\rho g = \frac{V}{3}\times 13.5\ g + \frac{2V}{3}\times 1.2\ g$

$\rho = \frac{13.5+1.2}{3} = \frac{15.9}{3} = 5.3$

92) c Condition of no difference in length of two rods with rise of temperature is:

$L_1\alpha_1 = L_2\alpha_2$

$L_2 = \frac{L_1\alpha_1}{\alpha_2} = \frac{16\times18\times10^{-6}}{12\times10^{-6}} = 24$ cm

93) b In a closed vessel, $T = constant$

$\frac{P_1}{T_1} = \frac{P_2}{T_2}$

$\frac{P_1}{T_1} = \frac{\left(P_1+\frac{0.5}{100}P_1\right)}{T_1+2}$

$\frac{T_1+2}{T_1} = 1 + \frac{0.5}{100}$

$1 + \frac{2}{T_1} = 1 + \frac{0.5}{100}$

$\frac{2}{T_1} = \frac{0.5}{100}$

$T_1 = 400\ K = 127^\circ C$

94) b First overtone of closed pipe $P_1 = 3\left(\frac{v}{4l_1}\right)$

Second overtone of open pipe $P_2 = 3\left(\frac{v}{2l_2}\right)$

They are in resonance. So,

$3\left(\frac{v}{4l_1}\right) = 3\left(\frac{v}{2l_2}\right)$

$\frac{l_1}{l_2} = \frac{1}{2}$

95) c $\mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin(90^\circ - i)} = \tan i\quad [\because i + r = 90^\circ]$

$\tan i = \sqrt{3}$

$i = 60^{\circ}$

96) b The position of n^{th} minima in the diffraction pattern is:

$x_n = \frac{n\lambda D}{d}$

$x_3 - x_1 = (3-1)\frac{\lambda D}{d} = \frac{2\lambda D}{d}$

$d = \frac{2\lambda D}{x_3 - x_1} = \frac{2\times 0.50\times 6000\times 10^{-10}}{3\times 10^{-3}} = 2\times 10^{-4}$ m

97) d $r = \frac{d}{2} = \frac{4.4}{2} = 2.2\, m$

Charge on the sphere, $q = \sigma \times 4\pi r^2 = 60\times 10^{-6}\times 4\times \frac{22}{7}\times (2.2)^2 = 3.7\times 10^{-3}$ C

98) d 6 Ω and 6 Ω are in series, so effective resistance is 12 Ω which is in parallel with 3 Ω, so

$\frac{1}{R} = \frac{1}{3} + \frac{1}{12}$

$R = \frac{36}{15}$

$\therefore I = \frac{V}{R} = \frac{4.8\times 15}{36} = 2$ A

99) a Force per unit length, $\frac{F}{l} = \frac{\mu_0}{4\pi}.\frac{2i_1 i_2}{a}$

$F = \frac{\mu_0}{4\pi}.\frac{2i_1 i_2}{a}\times l = 10^{-7}\times \frac{2\times 10\times 2}{(10\times 10^{-2})}\times 2 = 8\times 10^{-5}$ N

100) c For Balmer series, $n_1 = 2, n_2 = 3$ for 1st line and $n_2 = 4$ for second line.

$\frac{\lambda_1}{\lambda_2} = \frac{\left(\frac{1}{2^2} - \frac{1}{4^2}\right)}{\left(\frac{1}{2^2} - \frac{1}{3^2}\right)} = \frac{3/16}{5/36} = \frac{27}{20}$

$\lambda_2 = \frac{20}{27}\lambda_1 = \frac{20}{27}\times 6561 = 4860$ Å

❖❖❖❖ Thank You!!! ❖❖❖❖