INSTITUTE OF ENGINEERING

MODEL ENTRANCE EXAM

(SET – 6)
Solutions

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Date : 2081/03/15
(June 29)

Duration : 2 hours
Time : 8 A.M. – 10 A.M.

SECTION – A (1 marks) (1*60 = 60)

1) c We have, $A = B \cap C$ and $B = C \cap A$
$A = (C \cap A) \cap C$
$\Rightarrow A = C \cap A = B$

2) b $A^2 - A + 2I = 0$
$A - A^2 = 2I$
$A(I - A) = 2I$
$A\left\{\frac{1}{2}(I - A)\right\} = I$
$A^{-1} = \frac{1}{2}(I - A)$

3) b In order to generate signals of 5 flags from 8 flags of different colours, we first select 5 flags from 8 flags. This can be done in $^8C_5$ ways. Now, we arrange selected 5 flags in 5! ways. Hence, the total number of signals = $^8C_5 \times 5!$

4) b $S = \frac{perimeter}{2} = \frac{a+b+c}{2} = \frac{8}{2} = 4\ cm$
Radius of inscribed circle = $\frac{40}{4} = 10\ cm$

5) b

6) b The two diameters intersect at (8, -2) which is the centre of circle. The circle passes through (6, 2).
$\therefore$ Radius = $\sqrt{(8-6)^2 + (-2-2)^2} = \sqrt{20}$

7) c $\lim_{n\to\infty} \frac{1^3+2^3+\cdots+n^3}{n^4} = \lim_{n\to\infty} \frac{\left\{\frac{n(n+1)}{2}\right\}^2}{n^4} = \lim_{n\to\infty} \frac{(n+1)^2}{4n^2} = \frac{1}{4}$

8) d Let r be the radius and V be the volume of the sphere. Then,
$V = \frac{4}{3}\pi r^3$
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$
$\frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$ $\left[\frac{dV}{dt} = \frac{dr}{dt} \text{ (given)}\right]$
$4\pi r^2 = 1$
$r = \frac{1}{2\sqrt{\pi}}$

9) a $I = \int \frac{1}{\sqrt{x^2+2}} d(x^2 + 2) = 2\sqrt{x^2 + 2} + c$

10) d We have, $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$
$\vec{a} = -\frac{8}{7}\vec{c}$
$\vec{a}$ and $\vec{c}$ are unlike parallel vectors.
Angle between $\vec{a}$ and $\vec{c}$ is π.

11) b Let $z = 0 + bi$, where $b > 0$. Then, z lies on y-axis.
So, $\arg(z) = \frac{\pi}{2}$

12) d Angle between lines is π/2. Since, they are line; one horizontal and other is vertical.

13) c Distance between planes = $\frac{3-\frac{5}{2}}{\sqrt{1+1+1}} = \frac{1}{2\sqrt{3}}$

14) c The function is continuous at R where $x - 5 \neq 0$.

15) a $\frac{dy}{dx} = -\frac{fx}{fy} = -\frac{2ax+2hy}{2hx+2by} = -\frac{ax\_hy}{hx+by}$

16) b Let α, β be the roots of the given equation. Then,
$\alpha + \beta = \frac{4+\sqrt{5}}{5+\sqrt{2}}$ and $\alpha\beta = \frac{8+2\sqrt{5}}{5+\sqrt{2}}$
Let H be the H.M. of α and β. Then,

$H = \frac{2\alpha\beta}{\alpha+\beta} = \frac{16+4\sqrt{5}}{4+\sqrt{5}} = 4$

17) b Total number of functions from A to B $= 2^3 = 8$

18) d We have, $4sin^2x - 8\sin x + 3 \leq 0$
$(2\sin x - 1)(2\sin x - 3) < 0$
$2\sin x - 1 \geq 0$ [$\because$ $2\sin x - 3 \leq 0$ for all x]
$\sin x \geq \frac{1}{2}$
$x \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

19) a We have, $y = Ax + A^3$
$\frac{dy}{dx} = A$
On eliminating A, we get,
$y = x\ dy.dx + \left(\frac{dy}{dx}\right)^3$
Clearly, it is a differential equation of degree 3.

20) b Clearly, Required mean $= \frac{n_{C_0}+n_{C_1}+\cdots+n_{C_n}}{n} = \frac{2^n}{n}$

21) a When brown ring complex dissolves in water, it dissociates to give $[Fe(H_2O)_5(NO)]^{2+}$ and $SO_4{}^{2-}$.
Here, NO is in +1 oxidation state and H_2O is neutral.
So, x+5(0)+1 = 2
$x = +1$

22) a Carbon monoxide has a triple covalent bond in which two are normal covalent bonds and one is a dative bond.

23) c Permanent hardness is caused by the presence of following salts in water:
- Calcium sulphate ($CaSO_4$) and Magnesium Sulphate ($MgSO_4$)
- Calcium chloride ($CaCl_2$) and Magnesium Chloride ($MgCl_2$)

24) c In this reaction, brown fumes of NO_2 are evolved.

25) d Inert gases do not support combustion.

26) a Smelting of an ore is done to reduce it to give metal.

27) d Excess of sodium hydroxide reacts with zinc to form sodium zincate along with liberation of hydrogen.
2 NaOH + Zn → Na_2ZnO_2 + H_2

28) b Atomic weight of Carbon = 12 mg
12 g of Carbon contains 6.023×10^{23} number of atoms.
Number of Carbon atoms present in 1×10^{-3} g of Carbon $= \frac{1\times10^{-3}}{12} \times 6.023 \times 10^{23}$
$= 0.502 \times 10^{20}$

29) d $n = 4; l = 0, 1, 2, 3$
$l = 3$ (d-orbital)
$m = -1$ to $+1$; $s = -\frac{1}{2}$ or $\frac{1}{2}$

30) d In a face-centred cubic (FCC) arrangement, the number of atoms per unit cell is 8.

31) c Value of enthalpy, internal energy and entropy depend on state and not path followed, so they are state functions. Work is not state function because its value depends on path followed.

32) c

33) d On warming with silver powder, chloroform is converted into acetylene.

34) b

35) c $[X] = [Force] \times [Density] = [MLT^{-2}] \times [ML^{-3}] = [M^2L^{-2}T^{-2}]$

36) b Action and reaction force always act in pairs and on different bodies.

37) d When the total external force acting on the system is zero, the velocity of centre of mass remains constant.
38) d Linear momentum is not conserved.
39) d The circular motion of a particle with constant speed is periodic but not simple harmonic motion as the particle repeats its motion after a regular interval of time but does not oscillate about a fixed point.
40) c In a given P-V diagram, pressure remains constant although volume increases. Hence, the process is an isobaric process.
41) b According to Wein's law,
$\lambda_m T = \text{constant}$
$\lambda_m \propto T^{-1}$
42) d The phenomenon of beats can take place for both longitudinal and transverse waves.
43) d Electric field at a point is continuous if there is no charge at that point, and the field is discontinuous if there is charge at that point.
44) d The force between two parallel current carrying wires is independent of the radii of the wires.
45) b
46) c $\beta = \frac{\lambda D}{d}$
If we replace yellow light with blue light, i.e., longer wavelength with shorter one, the fringe width decreases.
47) b $\lambda = \frac{h}{p} = \frac{h}{mv}$
If the velocity of the electron increases, de Broglie wavelength decreases.
48) d Electrons are the majority charge carriers in n-type semiconductors.
49) c
50) d
51) d
52) a
53) a
54) d
55) a
56) c
57) b
58) a
59) b
60) c

<u>SECTION – B (2 marks)</u> (2*40=80)

61) b We have,
$f(x) = \frac{x}{x+1} = 1 - \frac{1}{1+x}$
Let $x, y \in R$ be such that $f(x) = f(y)$. Then,
$f(x) = f(y)$
$1 - \frac{1}{1+x} = 1 - \frac{1}{1+y}$

$1 + x = 1 + y$
$x = y$
∴ f is one-one.
Clearly, f is not onto as f takes only values less than 1 i.e., Range of $f = (-\infty, 1) \neq$ co-domain of f.

62) d $|x| < 1$, so assume $x = 0.2$.
∴ $1 + 3 \times 0.2 + 6 \times 0.2^2 + 10 \times 0.2^3 = 1.92$
Now, put $x = 0.2$ in options and evaluate
a. $\frac{1}{(1-0.2)^2} = 1.56$
b. $\frac{1}{1-0.2} = 1.25$
c. $\frac{1}{(1+0.2)^2} = 0.96$
d. $\frac{1}{(1-0.2)^3} = 1.95 > 1.92$
Option (d) holds true.

63) c $(1+x)^{50} \times (1-x+x^2)^{50} = (1+x^3)^{\mathbf{50}} = \sum_{r=0}^{50} C(50,r)(x^3)^r$
For coefficient of x^{30}, $r = 10$
∴ coefficient of $x^{30} = C(50,10) = C(50, 50-10) = C(50,40)$

64) b $C_{r+1} + {}^nC_{r-1} + 2\,{}^nC_r = C_{r+1} + {}^nC_r + {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_{r+1} + {}^{n+!}C_r = {}^{n+2}C_{r+1}$

65) d Number divisible by 4 are 12, 24, 32, 44, 52.
Total numbers = $5 \times 5 = 25$
$P(E) = \frac{5}{25} = \frac{1}{5}$

66) b $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$
$\sin^{-1} x + \cos^{-1} x - 2\cos^{-1} x = \frac{\pi}{6}$
$\frac{\pi}{2} - 2\cos^{-1} x = \frac{\pi}{6}$
$2\cos^{-1} x = \frac{\pi}{3}$
$\cos^{-1} x = \frac{\pi}{6}$
$x = \frac{\sqrt{3}}{2}$

67) b Use method of rejection of options. Any pair of line perpendicular to given lines will have coefficient x^2 as 3, coefficient of y^2 as 5 and coefficient of $xy = +8$. So, options a and c are rejected. Now for remaining two options b and d, test whether (3, -2) satisfies them. Now with (3, -2), Option (b) $= 27 - 48 + 20 - 6 + 8 - 1 = 0$. So, b is the correct option.

68) d For $2x + 3y = 1$
$y = -\frac{2x}{3} + \frac{1}{3}$
$m = -\frac{2}{3}$ & $c = \frac{1}{3}$
Line touches parabola: $c = \frac{a}{m}$
$a = -\frac{2}{9}$
Length of L.R. $= 4a = \frac{8}{9}$

69) d Centre of ellipse → Origin
Focus $= (ae, 0) = \left(\sqrt{a^2 - b^2}, 0\right) = (3, 0)$
Equation of parabola is:
$y^2 = 4 \times 3x = 12x$

70) b If θ be the angle, then

$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{({a_1}^2+{b_1}^2+{c_1}^2)({a_2}^2+{b_2}^2+{c_2}^2)}} = \frac{6+4-10}{\sqrt{(50)(9)}} = 0$

i.e., $\theta = \frac{\pi}{2}$

71) c $x^y = e^{(x-y)}$

$\log x^y = \log e^{(x-y)}$

$y \log x = (x-y)\log e$

$y = \frac{x}{1+\log x}$

$y\log x - x + y = 0$

$\frac{dy}{dx} = \frac{-\log x}{(\log x + 1)^2}$

72) b $\left|\vec{a}+\vec{b}\right|^2 < \left|\vec{a}-\vec{b}\right|^2$

$4\vec{a}.\vec{b} < 0$

$ab\cos\theta < 0$

$\cos\theta < 0$

$\theta > 90^\circ$ i.e. obtuse angle.

73) c $I = \int \frac{x^5}{\sqrt{1+x^3}}dx$

Put $1 + x^3 = t^2$

Then, $3x^2\, dx = 2t\, dt$

$I = \int \frac{x^3x^2}{\sqrt{1+x^3}}dx = \frac{2}{3}\int \frac{(t^2-1)\, tdt}{t} = \frac{2}{3}\int(t^2-1)dt = \frac{2}{3}\left[\frac{t^3}{3} - t\right] + c$

$= \frac{2}{3}\left[\frac{(1+x^2)^{3/2}}{3} - (1+x^2)^{1/2}\right] + c$

74) a Let $y = x(1-x)^2 = x(1-2x+x^2) = (x-2x^2+x^3)$

$y' = 1 - 4x + 3x^2$ and $y'' = -4 + 6x$

For extreme values, $y' = 0 \Rightarrow 3x^2 - 4x + 1 = 0$

$x = 1, \frac{1}{3}$

For $x = 1, y'' = -4 + 6 = 2 > 0$

For $x = \frac{1}{3}, y'' = -4 + 6 \times \frac{1}{3} = -2 < 0$ (Max)

And $x = \frac{1}{3} \in [0, 2]$

$\therefore \mathrm{y_{max}} = \frac{1}{3}\left(1 - \frac{1}{3}\right)^2 = \frac{4}{27}$

75) a Solving $y = x$ and $y = x + \sin x$, we have,

$x = x + \sin x$

$\sin x = 0$

$x = 0, \pi$

Area $= \int_0^\pi (y_1 - y_2)dx = \int_0^\pi (x + \sin x - x)dx = \int_0^\pi \sin x\, dx = [-\cos x]_0^\pi$

$= [-\cos\pi + \cos 0] = 2$ sq. unts

76) a Time taken by body A, $\mathrm{t_1} = 5$ s

Acceleration of body A = $\mathrm{a_1}$

Time taken by body B, $\mathrm{t_2} = 5 - 2 = 3$ s

Acceleration of body B = $\mathrm{a_2}$

Distance covered by first body in 5th second after its start,

$\mathrm{S_5 = u + \frac{a_1}{2}(2t_1 - 1) = 0 + \frac{a_1}{2}(2 \times 5 - 1) = \frac{9}{2}a_1}$

Distance covered by the second body in 3rd second after its start,

$\mathrm{S_3 = u + \frac{a_2}{2}(2t_2 - 1) = 0 + \frac{a_2}{2}(2 \times 3 - 1) = \frac{5}{2}a_2}$

Since, $S_5 = S_3$

or, $\frac{9}{2}a_1 = \frac{5}{2}a_1$

$a_1 : a_2 = 5 : 9$

77) b Here, $u = 15\ ms^{-1}, h = 490\ m$

Time taken by the stone to reach the ground is:

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times490}{9.8}} = 10\ s$

78) d Here, Surface tension (S) $= 2.5 \times 10^{-2}\ Nm^{-1}$

$r = 6\ mm = 6 \times 10^{-3}\ m$

Excess pressure inside the soap bubble,

$P = \frac{4S}{r} = \frac{4\times2.5\times10^{-2}}{6\times10^{-3}} = 16.7\ Pa$

79) a As $V_T = V_0(1 + \gamma\Delta T)$

$\frac{V_T - V_0}{V_0} = \gamma\Delta T$

$\frac{0.24}{100} = 40\ \gamma$

$\gamma = \frac{0.24}{100\times40} = 6 \times 10^{-5}\ °C^{-1}$

Coefficient of linear expansion $\alpha = \frac{\gamma}{3} = \frac{6\times10^{-5}}{3} = 2 \times 10^{-5}\ °C^{-1}$

80) d Work done in isothermal process is:

$W = nRT \ln\frac{V_2}{V_1} = 1 \times 8.31 \times 310 \times \ln\frac{2V_1}{V_1} = 1.786 \times 10^3$

Amount of heat absorbed $= \frac{1.786\times10^3}{4.2}$ cal = 425.4 cal

81) c $T_m = 2\pi\sqrt{\frac{l}{g_m}}$ and $T_e = 2\pi\sqrt{\frac{l}{g_e}}$

$\frac{T_m}{T_e} = \sqrt{\frac{g_e}{g_m}}$

$T_m = \sqrt{\frac{g_e}{g_m}} \times T_e = \sqrt{\frac{9.8}{1.7}} \times 3.5 = 8.4\ s$

82) b The given equation of a wave is:

$y = 10 \sin\left(\frac{2\pi}{45}t + \alpha\right)$

At $t = 0, y = 5\ cm$

$5 = 10 \sin\alpha$

$\frac{1}{2} = \sin\alpha$

$\alpha = \frac{\pi}{6}$

Hence, the total phase at $t = 7.5$ s is:

$\phi = \frac{2\pi}{45} \times 7.5 + \alpha = \frac{\pi}{3} + \frac{\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$

83) a When a charged capacitor of capacitance C_1 is connected in parallel to an uncharged capacitor of capacitance C_2, the charge is shared between them till both attain the common potential which is given by:

$V' = \frac{q_1+q_2}{C_1+C_2} = \frac{C_1V+0}{C_1+C_2} = \frac{C_1V}{C_1+C_2}$

84) b In first case, $12 = E - 2r$ --- (1)

In second case, $15 = E + 3r$--- (2)

Subtract (2) from (1), we get, $r = \frac{3}{5}\ \Omega$

From (1),

$12 = E - 2 \times \frac{3}{5}$

$E = 12 + \frac{6}{5} = \frac{66}{5} = 13.2\ V$

85) c As, $B_V = \sqrt{3}\ B_H$

Also, $\tan\delta = \frac{B_V}{B_H} = \frac{\sqrt{3}B_H}{B_H} = \sqrt{3}$

$\delta = \tan^{-1}(\sqrt{3}) = 60^\circ$

86) b Current in the circuit, $I = \frac{V}{Z} = \frac{220}{44} = 5\ A$

Power dissipated in the circuit, $P = I^2R = 5^2 \times 22 = 550\ W$

87) a Here, $u = -40$ cm, $v = +80$ cm, $f = ?$

$\frac{1}{f} = \frac{1}{80} - \frac{1}{-40} = \frac{1+2}{80} = \frac{3}{80}$

Now,

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

$\frac{1}{f_2} = \frac{3}{80} - \frac{1}{25} = \frac{15-16}{400} = -\frac{1}{400}$

$f_2 = -400$ cm

88) a As $eV_0 = h(v - v_0)$

$V_0 = \frac{h(v-v_0)}{e} = \frac{6.63\times10^{-34}(8.2\times10^{14}-3.3\times10^{14})}{1.6\times10^{-19}} = 2\ V$

89) b $N_1V_1 = N_2V_2$

(NaOH) (Acid)

$\frac{1}{10} \times 25 = N_2 \times 20$

$N_2 = 0.125$

Strength = Normality × Eq. mass

Eq. mass = $\frac{7.875}{0.125} = 63$

90) d $CaCO_3 \xrightarrow{\Delta} CaO + CO_2$

Molar mass of $CaCO_3$ = 100 g/mol

10 g of 90% pure lime $= \frac{10\ g}{100\ g/mol} \times \frac{90}{100} = 0.09$ moles $CaCO_3$

0.09 moles of $CaCO_3$ on heating gives 0.09 moles of CO_2

At STP, 1 mole CO_2 = 22.4 L

At STP. 0.09 mole CO_2 = $0.09 \times 22.4 = 2.016$ L

91) c Millimoles of CH_3COOH = $0.1 \times 10 = 1$

Millimoles of CH_3COONa = $0.1 \times 20 = 2$

$p_H = p_{ka} + \log\frac{[\text{conjugate base}]}{[\text{acid}]} = 4.74 + \log\frac{2}{1} = 4.74 + 0.30 = 5.04$

92) a

Electronic Configuration	Group
$[Ne]3s^23p^3$	V
$[Ne]3s^23p^2$	IV
$[Ar]3d^{10}4s^24p^3$	V
$[Ne]3s^23p^1$	III

Since, ionization energy increases in a period and decreases in a group, $[Ne]3s^23p^3$ configuration has the highest ionization energy among the given elements.

93) b More the number of sigma C–H bonds at alpha position, more the number of hyper conjugating structures and more the stability. Hence, structure (b) will be the most stable carbocation.

94) b

95) d Number of equivalents of aluminium deposited $= \frac{4.5}{9} = 0.5$

Number of equivalents of H_2 will also be 0.5.

Volume of H_2 gas at STP = Number of equivalents × Equivalent volume
$= 0.5 \times 11.2$
$= 5.6$ L

96) c $k = \frac{2.303}{t}\log\frac{C_0}{C_t} = \frac{2.303}{40}\log\frac{0.1}{0.025} = \frac{2.303}{40} \times \log 4 = \frac{2.303\times0.6021}{40}$
Rate = k[X]
$= \frac{2.303\times0.6021}{40} \times 0.01 = 3.47 \times 10^{-4}$ M/min

97) b For something to be fundamental, it has to be an essential part of something. In this instance, the fundamental shift means that the rules around shopping have changed significantly; therefore, 'major' is the best option.

98) d In the second paragraph we are told that 'Whilst there were concerns about online trading in the early days, this has obviously declined now and as confidence in the internet grows, so too does online shopping.' This helps us to see that people were less confident about buying goods on the internet initially, but their confidence has now grown thus making option D correct. We do not have direct evidence to support any of the other three statements.

99) a The second paragraph tells us that people are busy and that they have less time to visit the shops. We are also told that shopping online allows consumers to shop when it suits them - hence it is an way to fit more into their busy lives.

100) a Option A is false as the first paragraph tells us about the fundamental shift in shopping patterns in the last three years. Option B is true - 'a trip to the shops is still regarded by many as an enjoyable past-time'. Options C and D are also true - 'consumers can shop when it suits them and can also use price comparison and review websites to ensure they are getting the best deal.

❖❖❖❖ Thank You!!! ❖❖❖❖