INSTITUTE OF ENGINEERING

MODEL ENTRANCE EXAM

(Beats Test Series – Day 1)
Solutions

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Date : 2081/05/01
(August 07)

Duration : 2 hours
Time : 8 A.M. – 10 A.M.

<u>SECTION – A</u> (1 marks) (1*60 = 60)

1) a
2) c
3) a
4) c
5) b
6) c
7) c
8) b
9) a
10) c
11) b
12) c
13) d n(A ∪ B) is minimum when $n(A \cap B)$ is maximum i.e., $A \subset B$ i.e., $(A \cap B) = 4$
Now, $\mathrm{n(A \cup B)} = n(A) + n(B) - n(A \cap B) = 4 + 7 - 4 = 7$
14) c $\mathrm{A'A} = \begin{bmatrix} -\mathrm{i} & 0 \\ 0 & \mathrm{i} \end{bmatrix}\begin{bmatrix} -\mathrm{i} & 0 \\ 0 & \mathrm{i} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = -\mathrm{I}$
15) c $(\mathrm{i})^{\mathrm{n}+4} = i^n . i^4 = i^n (i^2)^2 = i^n (-1)^2 = i^n$
16) d Let $y = \sqrt{6 + \sqrt{6} + \sqrt{6} + \cdots} + \infty$
$y^2 = 6 + \sqrt{6 + \sqrt{6} + \cdots} + \infty$
$y^2 = 6 + y$
On solving, $y = 3, -2$
17) c There are 5 ways of entering into a campus. Since, a man has to come out through a different gate, so number of ways he can come out is 4.
Total number of ways $= 5 \times 4 = 20$
18) c The $(r+1)^{th}$ term of the series $(1 + x^2)^{10}$ is:
$t_{r+1} = C(10, r)(x^2)^r = C(10, r)x^{2r}$
To get the coefficient of x^8, put $2r = 4 \Rightarrow r = 2$
Thus, required coefficient of x^8 is $C(10, 2) = \frac{10!}{2!8!} = 45$
19) d $2\cos^2\theta + 3\sin\theta = 0$
$2 - 2\sin^2\theta + 3\sin\theta = 0$
$2\sin^2\theta - 3\sin\theta - 2 = 0$
$(2\sin\theta + 1)(\sin\theta - 2) = 0$
i.e., $\sin\theta = -\frac{1}{2} = \sin\left(\pi + \frac{\pi}{6}\right)$
$\theta = \frac{7\pi}{6}$
20) a Put $\sin^{-1}\left(\frac{2}{5}\right) = \theta \Rightarrow \sin\theta = \frac{2}{5}$
$\sin\left\{3\sin^{-1}\left(\frac{2}{5}\right)\right\} = \sin 3\theta = 3\sin\theta - 4sin^3\theta = 3\left(\frac{2}{5}\right) - 4\left(\frac{2}{5}\right)^3 = \frac{118}{125}$
21) a $r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = 4R\sin\frac{60^\circ}{2}\sin\frac{60^\circ}{2}\sin\frac{60^\circ}{2} = 4R\frac{1}{2}.\frac{1}{2}.\frac{1}{2}$
$R = 2r$
22) b The equation of the line is: $\frac{x}{h} + \frac{y}{k} = 1$
It passes through (3, 3) i.e., $\frac{3}{h} + \frac{3}{k} = 1 \Rightarrow \frac{1}{h} + \frac{1}{k} = \frac{1}{3}$

23) d Comparing with homogenous equation $ax^2 + 2hxy + by^2 = 0$,
$a = 1, h = \frac{7}{2}, b = -1\backslash$
Here, $a + b = 1 - 1 = 0$.
Hence, lines are perpendicular.

24) c Length of latus rectum = 1/2 (Length of major axis)
$\frac{2b^2}{a^2} = \frac{1}{2}(2a)$
$2b^2 = a^2$
$2a^2(1 - e^2) = a^2$
$1 - e^2 = \frac{1}{2}$
$e^2 = \frac{1}{2}$
$e = \frac{1}{\sqrt{2}}$

25) d Equation of the hyperbola is:
$\frac{x^2}{9} - \frac{y^2}{16} = 1$
Length of latus rectum $= \frac{2b^2}{a} = \frac{2.16}{3} = \frac{32}{3}$

26) b Ratio $= \frac{Ax_1+By_1+Cz_1+D}{Ax_2+By_2+Cz_2+D} = \frac{2.2+1.-3+1.1-7}{2.3+1.-4+1.-5-7} = -\frac{1}{2}$

27) c $y = a\sin(5x + c)$
$\frac{dy}{dx} = 5\,\mathrm{a}\cos(5x + c)$
$\frac{d^2y}{dx^2} = -25a\sin(5x + c) = -25y$

28) d Two vector are orthogonal if $\vec{a}.\vec{b} = 0$
$\therefore \left(\vec{\imath} - 2\vec{\jmath} + 3\vec{k}\right).\left(2\vec{\imath} + 7\vec{\jmath} + m\vec{k}\right) = 0$
$2 - 14 + 3m = 0$
$m = 4$

29) b Quartile deviatoon $= \frac{Q_3 - Q_1}{2} = \frac{33-23}{2} = \frac{10}{2} = 5$

30) c $\lim_{x\to a} \frac{\sqrt[3]{x} - \sqrt[3]{a}}{\sqrt{x} - \sqrt{a}} = \lim_{x\to a} \frac{x^{1/3} - a^{1/3}}{x^{1/2} - a^{1/2}} = \frac{\left(\frac{1}{3}\right)}{\left(\frac{1}{2}\right)}.a^{\frac{1}{3}-\frac{1}{2}} = \frac{2}{3}a^{-1/6} = \frac{2}{3a^{1/6}}$

31) c $x^2 + y^2 - 4 = 0$
$\frac{dy}{dx} = -\frac{2x+0-0}{0+2y-0} = -\frac{x}{y}$

32) a $I = \int \frac{\sin^4 x}{\cos^6 x}\,\mathrm{dx} = \int tan^4\mathrm{x}.sec^2\mathrm{x}\,\mathrm{dx}$
Put $y = \tan x$
$dy = sec^2x\,dx = \int y^4 dy = \frac{y^5}{5} + c = \frac{tan^5x}{5} + c$

33) c

34) c

35) b Hoffmann bromamide degradation reaction.

36) d One molecule of CO contains one oxygen atom.
6.02×10^{24} molecules of CO contain 6.02×10^{24} oxygen atoms.
6.02×10^{23} atoms of oxygen = 1 g-atom of oxygen
6.02×10^{24} atoms of oxygen = 10 g-atom of oxygen.

37) d

38) d Fluorine has the smallest atomic size and greatest electronegativity in the periodic table. Therefore, it always shows a negative oxidation state.

39) d

40) a

41) b The standard reduction potential of zinc is negative (-0.76 V). Zn has a high tendency to lose electrons and hence, it forms an anode in an electrochemical cell.
$Zn \rightarrow Zn^{2+} + 2e^-$

42) a Size $\propto \frac{1}{\text{Effective Nuclear Charge}}$

43) c

44) a

45) c Glass being a mixture of sodium and calcium silicates react with hydro fluoric acid forming sodium and calcium fluorosilicates respectivley.

46) c Cyanide process
$Ag_2S + KCN \rightleftharpoons 2K[Ag(CN)_2] + K_2S$

47) d $\tan 30^\circ = \frac{A_y}{A_x}$
$A_y = 3 \times \frac{1}{\sqrt{3}} = \sqrt{3}$

48) a Cooking pot must have low specific heat capacity and high thermal conductivity so that minimum heat is lost.

49) a independent to change in pressure

50) a Velocity decreases due to decreases in wavelength.

51) b $E = \frac{V}{d} = \frac{Q}{C.d}$
On introducing dielectric slab, capacitance increases. So, electric field intensity decreases.

52) b Stream of proton act as parallel conductor carrying current in same direction, so they attract each other.

53) a To emit x-ray, energy difference between two energy level must lie in x-ray region.

54) c Two angle of projections to have same range are θ and $(90^\circ - \theta)$.
$h_1 + h_2 = \frac{u^2 sin^2\theta}{2g} + \frac{u^2 sin^2(90^\circ - \theta)}{2g} = \frac{u^2}{2g}$

55) c $\frac{T\prime}{T} = \left(\frac{R\prime}{R}\right)^{3/2} = \left(\frac{4R}{R}\right)^{3/2} = 8$
$T' = 8T$

56) d $\mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{15}{6+4} = 1.5$

57) b $y_1 = 4\sin 100\pi t$ and $y_2 = 3\cos 100\pi t = 3\sin(90^\circ + 100\pi t)$
$a_R = \sqrt{3^2 + 4^2} = 5$ units

58) c $\frac{v'}{v} = (n)^{2/3}$
$v' = (64)^{2/3} v = 16v$

59) d $R_{eq} = \frac{4 \times 2}{4+2} = \frac{4}{3}\ \Omega$

60) a For He, $2eV = \frac{1}{2} 4m{v_{He}}^2$
$v_{He} = \sqrt{\frac{4eV}{4m}} = \sqrt{\frac{eV}{m}}$ --- (i)
For H, $eV = \frac{1}{2} m{v_H}^2$
$v_H = \sqrt{\frac{2eV}{m}}$ --- (ii)
$\frac{v_{He}}{v_H} = \frac{1}{\sqrt{2}}$

SECTION – B (2 marks) (2*40=80)

61) c $f(x)$ is real
i.e., $x^2 - 5x + 6 > 0 \Leftrightarrow (x-2)(x-3) > 0$
$x < 2$ or $3 < x$
$-\infty < x < 2$ or $3 < x < \infty$
i.e., $x \in (-\infty, 2) \cup (3, \infty)$

62) b $C_1 \to C_2 - p(C_2 + C_3)$
$$\begin{vmatrix} 0 & x & y \\ 0 & y & z \\ -(p^2x + 2py + z) & xp + y & yp + z \end{vmatrix} = 0$$
$-(p^2x + 2py + z)(xz - y^2) = 0$
Either, $-(p^2x + 2py + z) = 0$
Or, $(xz - y^2) = 0$
$y^2 = xz$
i.e., x, y, z are in G.P.

63) c $(1 + 2 + 3 + \cdots + n) = \frac{1}{5}(1^2 + 2^2 + 3^2 + \cdots + n^2)$
$\frac{n(n+1)}{2} = \frac{1}{5}\frac{n(n+1)(2n+1)}{6}$
$2n + 1 = 15$
$n = 7$

64) a We have, $log_e(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \cdots \infty$
Putting $x = 1$
$log_e 2 = \left(1 - \frac{1}{2}\right)\left(\frac{1}{3} - \frac{1}{4}\right)\left(\frac{1}{5} - \frac{1}{6}\right) + \cdots + \infty$
$= \left(\frac{2-1}{1.2}\right) + \left(\frac{4-3}{3.4}\right) + \left(\frac{6-5}{5.6}\right) + \cdots + \infty$
$= \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \cdots + \infty$

65) b $\cot\frac{A}{2}\cot\frac{B}{2} = \sqrt{\frac{s(s-a)}{(s-b)(s-c)}}.\sqrt{\frac{s(s-b)}{(s-c)(s-a)}} = \frac{s}{s-c} = \frac{2s}{2s-2c} = \frac{4c}{2c} = 2$
$(\because a + b + c = 3c \Rightarrow a + b + c = 4c \Rightarrow 2s = 4c)$

66) c Product of two natural numbers is divisible by 7 if and only if at least one of the number is divisible by 7.
Required probability = 1 – P(none of the numbers is divisible by 7)
$= 1 - \frac{C(14,2)}{C(100,2)} = \frac{4859}{4950}$

67) c On analyzing each options, option (d) cannot be possible as it is a circle.
For a second degree equation, we have
$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$
From option (a), $\Delta \neq 0$
From option (b), $\Delta \neq 0$
For option (c), $\Delta = 0$
So, option (c) represents a pair of straight lines.

68) a Circles (S_1) : $x^2 + y^2 - 9 = 0$
Centre (C_1) : (0, 0) and $r_1 = 3$
Circle (S_2) : $x^2 + y^2 + 2ax + 2y + 1 = 0$
Centre (C_2) : (-a, -1) and $r_2 = \sqrt{a^2 + 1 - 1} = a$
When the circle touches externally, $C_1C_2 = r_1 + r_2$

i.e., $\sqrt{a^2+1} = 3 + a$
$a^2 + 1 = 9 + 6a + a^2$
$a = -\frac{4}{3}$

69) d Equation of normal to the parabola $y^2 = 4ax$ at point $(at^2, 2at)$ is:
$y = -tx + 2at + at^3$ --- (i)
Point $(a{t_1}^2, 2at_1)$ lies on (i) i.e.,
$2at_1 = -t(a{t_1}^2) + 2at + at^3$
$2at(t_1 - t) = -at({t_1}^2 - t^2)$
$2 = -t(t_1 + t) \qquad (t_1 \neq t)$
$-\frac{2}{t} = t_1 + t$
$t_1 = -\frac{2}{t} - t$

70) b Plane through intersection of $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ is:
$(x + 2y + 3z - 4) + \lambda(2x + y - z + 5) = 0$
i.e., $(1 + 2\lambda)x + (2 + \lambda)y + (3 - \lambda)z + (5\lambda - 4) = 0$ --- (i)
Since this plane is normal to $5x + 3y + 6z + 8 = 0$
$(1 + 2\lambda).5 + (2 + \lambda).3 + (3 - \lambda).6 = 0$
$\lambda = -\frac{29}{7}$
Substituting the value of λ in (i), $51x + 15y - 8z + 173 = 0$.

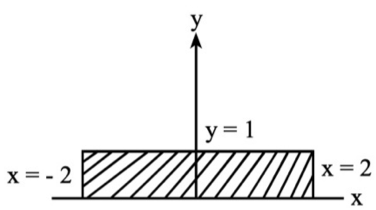
71) d First vector : $\sqrt{3}(\vec{a} \times \vec{b})$
Second vector : $\vec{b} - (\vec{a}.\vec{b})\vec{a}$
$\vec{a} \times \vec{b}$ is a vector perpendicular to both $\vec{a}$ and $\vec{b}$.
The vector $\vec{b} - (\vec{a}.\vec{b})\vec{a}$ is perpendicular to $\vec{a}$, as it is the projection of $\vec{b}$ onto the plane perpendicular to $\vec{a}$.
Since, $\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, it is also perpendicular to any vector in the plane containing $\vec{a}$ and $\vec{b}$, including $\vec{b} - (\vec{a}.\vec{b})\vec{a}$.
Thus, the angle between given vectors is 90° or π/2.

72) a Let $y = \lim_{x\_0^+} x^x$
$\log y = \lim_{x\_0^+} x \log x = \lim_{x\_0^+} \frac{\log x}{(1/x)} = \lim_{x\_0^+} \frac{\frac{1}{x}}{\left(-\frac{1}{x^2}\right)} = \lim_{x\_0^+}(-x^2) = 0$
$y = e^0 = 1$

73) a $\tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1} x$
$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1} x$
$\frac{d \tan^{-1}\left(\frac{2x}{1-x^2}\right)}{d \sin^{-1}\left(\frac{2x}{1+x^2}\right)} = \frac{d(2\tan^{-1} x)}{d(2\tan^{-1} x)} = 1$

74) a $\int_0^2 \frac{1-x}{1+x} dx = -\int_0^2 \frac{x-1}{1+x} dx = -\int_0^2 \frac{(x+1-2)}{1+x} dx = -\int_0^2 \frac{x+1}{1+x} dx + 2\int_0^2 \frac{1}{1+x} dx$
$= [x]_0^2 + [2\log(x+1)]_0^2 = 2\log 3 - 2$

75) c



Required area (A) $= \int_{-2}^{2} y\, dx = \int_{-2}^{2} 1\, dx = |x|_{-2}^{2} = (2-(-2)) = 4$ sq. units

76) c Compound B is a secondary alcohol since it gives blue colour in Victor Meyer test and secondary alcohol are obtained by the action of CH_3MgI on an aldehyde other then formaldehyde.

77) c $CH_3COOH \xrightarrow{Li/AlH_4} CH_3CH_2OH \xrightarrow{PCl_5} CH_3CH_2Cl \xrightarrow{alc.KOH} CH_2 = CH_2$

78) b $\Delta n_g = 2 - 4 = -2$

$\Delta H = \Delta U + \Delta n_g RT$

$\Delta U = \Delta H - \Delta n_g RT = [-92.38] - [(-2) \times 8.314 \times 10^{-3} \times 298]$

$= -92.38 + 4.955 = -87.42$ kJ

79) c $HQ \rightleftharpoons H^+ + Q^-$

$[H^+] = \sqrt{K_\alpha C}$ (By Ostwald's dilution law)

$10^{-3} = \sqrt{K_\alpha \times 0.1}$

$10^{-6} = K_\alpha \times 0.1$

$K_\alpha = 10^{-5}$

80) b Rate (H_2) $= \frac{50}{20} = 2.5$

Rate (O_2) $= \frac{40}{t}$

$\frac{r_{H_2}}{r_{O_2}} = \frac{2.5}{40/t} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}} = 4$

$2.5t = 160$

$t = 64$ min

81) b $N = \frac{W(gm) \times 1000}{V \times Eq.wt.}$

1500 mL of 0.1 N HCl = 150 mL (N)

$1 = \frac{W \times 1000}{150 \times 40}$

W = 6 gm

82) c $P_4O_6 + 6H_2O \rightarrow 3H_3PO_3 + PH_3$

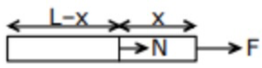
(Hot)

83) b In the given molecules we can see that the anion attached to each molecule is the same i.e. chlorine but cations are different. And the order of electronegativity of the cations is as: Na < Li. So, the increasing order of the covalent character will be $NaCl < LiCl < BeCl_2$.

84) b $x = \frac{1}{2}a(10)^2 = 50a$

$y = \frac{1}{2}a(20)^2 = 150a = 3x$

85) d



$a = \frac{F}{M}$

At P, $T = Ma = \left(\frac{M}{l}\right)(l-x)\frac{F}{M} = \left(\frac{l-x}{l}\right)F$

86) a $\frac{2T_1}{2T_2} = \frac{\rho_1 g h_1 r}{\rho_2 g h_2 r}$

$\frac{h_1}{h_2} = \frac{T_1}{T_2} \times \frac{\rho_2}{\rho_1} = \frac{6}{5} \times \frac{0.8}{0.4} = \frac{12}{5}$

87) b $l_b \alpha_b = l_i \alpha_i$

$\frac{l_b}{l_i} = \frac{\alpha_i}{\alpha_b} = \frac{\gamma_i}{\gamma_b} = \frac{36 \times 10^{-6}}{54 \times 10^{-6}} = \frac{2}{3}$

88) d $\frac{f'}{f} = \frac{d}{d'} = \frac{0.9}{0.93} = \frac{30}{31}$

% change $= \left(\frac{f'}{f} - 1\right) \times 100\%$

$= \left(\frac{30}{31} - 1\right) \times 100\% = -3.2\%$

89) c Apparent depth $= \frac{t}{2\mu_1} + \frac{t}{2\mu_2} = \frac{t}{2}\left(\frac{\mu_2+\mu_1}{\mu_1\mu_2}\right)$

90) c $w = \frac{f_r - f_b}{f_y} = \frac{21.4-20}{20.5} = \frac{14}{205}$

91) c Diagonal (d) $= \sqrt{3}b$

Distance of centre from vertex (r) $= \frac{d}{2} = \frac{\sqrt{3}b}{2}$

$\text{P.E.} = 8.\frac{(-q)}{4\pi\varepsilon_0 r}q = \frac{-8q^2\times 2}{4\pi\varepsilon_0\sqrt{3}b} = -\frac{4q^2}{\sqrt{3}\pi\varepsilon_0 b}$

92) d $R_1 : R_2 : R_3 = \frac{\rho l_1}{A_1} : \frac{\rho l_2}{A_2} : \frac{\rho l_3}{A_3}$

$= \frac{{l_1}^2}{v_1} : \frac{{l_2}^2}{v_2} : \frac{{l_3}^2}{v_3}$

$= \frac{{l_1}^2}{m_1} : \frac{{l_2}^2}{m_2} : \frac{{l_3}^2}{m_3}$

$= \frac{3^2}{1} : \frac{2^2}{2} : \frac{1^2}{3} = 9 : 2 : \frac{1}{3} = 27 : 6 : 1$

93) b $R = \frac{v^2}{P} = \frac{6^2}{6} = 6\ \Omega$

$P' = I^2 R = \left(\frac{R}{R+r}\right)^2 \times R = \left(\frac{6}{6+2}\right)^2 \times 6 = \frac{27}{8}$ W

94) c $I_g(R+G) = (I - I_g)S$

$R = (10 - 0.04)\frac{0.06}{0.04} - 10 = 4.94\ \Omega$

95) b $\lambda_1 = \frac{h}{\sqrt{2mE_1}}$

$E_1 = \frac{h^2}{2m{\lambda_1}^2}$ --- (i)

$\lambda_2 = \frac{h}{\sqrt{2mE_2}}$

$E_2 = \frac{h^2}{2m{\lambda_2}^2}$ --- (i)

$\frac{E_2}{E_1} = \left(\frac{\lambda_1}{\lambda_2}\right)^2 = \left(\frac{10^{-10}}{0.5\times 10^{-10}}\right)^2 = 4$

$E_2 = 4E_1$

Energy added $= E_2 - E_1 = 4E_1 - E_1 = 3E_1$

96) d Fraction left, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}} = \left(\frac{1}{2}\right)^{\frac{8}{4}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Fraction decay $1 - \frac{N}{N_0} = 1 - \frac{1}{4} = \frac{3}{4}$

97) a

98) b

99) b

100) d

❖❖❖❖ Thank You!!! ❖❖❖❖