INSTITUTE OF ENGINEERING

MODEL ENTRANCE EXAM

(Beats Test Series – Day 2)
Solutions

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Date : 2081/05/02 (August 18)

Duration : 2 hours
Time : 8 A.M. – 10 A.M.

<u>SECTION – A</u> (1 marks) (1*60 = 60)

1) b
2) a
3) c
4) b
5) a
6) c
7) a
8) d
9) c
10) a
11) c
12) c
13) a $(1+2i)^2 = 1+4i-4 = -3+4i$
Here, we see 4 is the imaginary part.
14) a $\mathrm{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathrm{I}$
$\mathrm{A}^9 = (\mathrm{I})^9 = \mathrm{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
15) c Multiplying both numerator and denominator by ω, we get
$\frac{a\omega+b\omega^2+c\omega^3}{\omega(a\omega+b\omega^2+c)} = \frac{1}{\omega}$ $\because (\omega^3 = 1)$
$= \frac{\omega^3}{\omega} = \omega^2$
16) b $\alpha+\beta = -p, \alpha\beta = q$
$\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = -p^3+3pq = 3pq-p^3$
17) a $x^{1/2}.x^{1/4}.x^{1/8}\ldots\infty = x^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots}$
$S_\infty = \frac{a}{1-r} = \frac{1/2}{1-1/2} = 1$
$\therefore x^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots} = x^1 = x$
18) b Here, 4 places are to be filled by 3 letters. So, total arrangement $= 4^3 = 64$
19) b $\tan 3x = 1 = \tan\frac{\pi}{4}$
$3x = n\pi + \frac{\pi}{4}$
$x = \frac{n\pi}{3} + \frac{\pi}{12}$
20) d Let $\cot^{-1}\mathrm{x} = \mathrm{A} \Rightarrow x = \cot A$
We have, $\sin A = \frac{p}{h} = \frac{1}{\sqrt{x^2+1}}$
$\sin(\cot^{-1} A) = (1+x^2)^{-1/2}$
21) c Angles are in the ratio $1:2:3$ i.e., $A = 30^\circ, B = 60^\circ, C = 90^\circ$
Using sine law, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
$\frac{a}{\sin 30^\circ} = \frac{b}{\sin 60^\circ} = \frac{c}{\sin 90^\circ}$
$\frac{a}{1/2} = \frac{b}{\sqrt{3}/2} = \frac{c}{1}$
$\mathrm{a:b:c} = 1:\sqrt{3}:2$
22) a Lines $3x-y=2, 5x+ay=3$ and $2x+y=3$ are concurrent. Then

$$\begin{vmatrix} 3 & -1 & -2 \\ 5 & a & -3 \\ 2 & 1 & -3 \end{vmatrix} = 0$$

$3(-3a+3)+1(-15+6)-2(5-2a)=0 \Rightarrow a=2$

23) a Comparing $x^2+y^2+2x+4y+k=0$ with $x^2+y^2+2gx+2fy+c=0$,
$g=1, f=2, c=k$
Equation will represent real circle if $g^2+f^2-c>0$
$1+4-k>0$
$5-k>0$
$5>k$
$\therefore k<5$

24) b For parabola $y^2=4x$
Focus (S_1) = (a, 0) = (1, 0)
For $x^2=-4y$
Focus(S_2) = (0, -a) = (0, -1)
On joining S_1 and S_2, we have,
$y-0=\frac{-1-0}{0-1}(x-1)$
$\Rightarrow x-y-1=0$

25) c Let $a^2=36, b^2=4$. Then eccentricity of the ellipse (e) = $\frac{36-4}{36}=\frac{32}{36}=\frac{8}{9}$
$e=\frac{2\sqrt{2}}{3}$
Equation of dielectrices are:
$x=\pm\frac{a}{e}=\pm\frac{6\times3}{2\sqrt{2}}=\pm\frac{9}{\sqrt{2}}$
$\sqrt{2}x\pm9=0$

26) b Length of perpendiicular (p) = $\pm\frac{3.2-4.3+5.4+2}{\sqrt{(3)^2+(-4)^2+5^2}}=\pm\frac{16}{5\sqrt{2}}=\frac{8\sqrt{2}}{5}$

27) b Total ball = $9+7+4=20$
$p(B)=\frac{4}{20}=\frac{1}{5}$
$\therefore P(\bar{B})=1-\frac{1}{5}=\frac{4}{5}$

28) a $\lim_{x\to\infty}\frac{\sin x}{x}=\lim_{x\to\infty}\frac{1}{x}.\lim_{x\to\infty}\sin x$
$=0.\lim_{x\to\infty}\sin x=0$

29) a $f'(x)=0$
$2x+4=0$
$x=-2$
Maximum value $f(-2)=(-2)^2+4(-2)+5=1$

30) c $\int uv\,dx=u\int v\,dx-\int\left(\frac{du}{dx}\int vdx\right)dx$
$\int_1^0 xe^x dx=[xe^x-e^x]_1^0=(0-1)-(e-e)=-1$

31) d Given differential equation is:
$x\,dy=y\,dx$
$\frac{dy}{y}=\frac{dx}{x}$
$\int\frac{dy}{y}=\int\frac{dx}{x}$
$log_e y=log_e x+log_e c$
$y=cx$, which is a straight line.

32) b $\sin\theta=\frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}=\frac{8}{10}=\frac{4}{5}$

$\cos\theta = \sqrt{1 - sin^2\theta} = \pm\frac{3}{5}$
$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta = 10\left(\pm\frac{3}{5}\right) = \pm 6$
$|\vec{a}.\vec{b}| = 6$

33) b Impulse = $F\,\Delta t = [MLT^{-2}][T] = [MLT^{-1}]$

34) c Horizontal component of velocity is taken as uniform.

35) c The apparent weight should be 40% of the weight of the fireman. i.e., $\frac{40}{100}Mg = M(g - a)$
$\Rightarrow a = g - \frac{2}{5}g = \frac{3}{5}g$

36) b Surface Energy (E) = S.T. × Surface area
$= S \times 4\pi r^2$
i.e., $E \propto r^2$

37) c Natural convection requires gravitational field.

38) a

39) d

40) b Intensity of electric field between the plates of charged condenser $= \frac{\sigma}{\epsilon_0} = \frac{q}{A\epsilon_0}$ $\because (\sigma = \frac{q}{A})$

41) d Magnetic force on charged particle is given as: $F = BqV\sin\theta$.
Here, $\theta = 0^\circ$. So, $F = 0$.

42) c Self inductance of circular coil having N turns is given by:
$L = \mu_0 N^2 A$
$L \propto N^2$

43) d $\mu = \frac{1}{\sin C}$. Since, μ decreases with increase in λ. Hence, C is minimum for blue colour, which has smallest wavelength.

44) b Spherical air bubble in water acts as concave lens when the angle of incidence is less than the critical angle and a convex mirror when the angle of incidence is more than critical angle.

45) d Emission will take place from higher value of n to lower value of n.
Frequency emmitted by H-like atom, $v = RC\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)$
For $n_1 = 2$ and $n_2 = 1$, it has highest frequency.

46) c Charge carriers in p-type crystals are holes.

47) a When an electron jumps from a lower to higher orbit, a photon gets absorbed during this process which results in the absorption of emergy.

48) d For sp hybidization, it has linear shape (180˚).
For sp^2 hybridization, it has trigonal planar geometry (120˚).
For sp^3 hybridization, it has tetrahedral shape (109˚).

49) d $HClO_4$
$1 + x + 4(-2) = 0$
$x = +7$

50) d Molecular crystals are those solids that are held together by Van der Waals forces.
e.g., Hydrogen, Iodine, dry ice.
Sodium chloride (NaCl) is an ionic solid having cations and anions (Na^+ and Cl^-). Hence, it is an example of an ionic crystal.

51) b $K_p = K_c.(RT)^{\Delta n}$
$\Delta n = 2 - 4 = -2$
$K_p = K_c(RT)^{-2}$
$K_p < K_c$

52) b While many enthalpies of formation are negative (exothermic), there are exceptions. For example, the formation of certain compounds like nitrogen oxides or carbon sulfide can have a positive enthalpy of formation.

53) b Fluorine although have the highest electronegativity due to its very small size, effective inter electronic repulsions are observed which bring down its electron affinity.

54) b Iron is more easily oxidized than copper, therefore it is used in recovvering copper from copper sulphate solution.

55) b $4Na + 2O_2 \rightarrow 2Na_2O_2$
Sodium peroxide

56) c Bell Metal : Cu + Sn
German Silver : Cu + Zn + Ni
Gun Metal : Cu + Sn + Zn + Pb
Rose Metal : Bi + Pb + Sn
Solder : Pb + Sn
Stainless metal : Fe + Cr + Ni + C
Brass : Cu + Zn
Bronze : Cu and Sn
Duralumin : Al + Cu + Mn + Mg
Pewter : Sn + Cu + Pb
Fernico : Fe + Ni + Co

57) c $2Hg + O_3 \rightarrow Hg_2O + O_2$

58) b

59) a $C_6H_5OH + Zn \rightarrow C_6H_6 + ZnO$

60) b Trichloroacetic acid will not undergo HVZ reaction due to the absence of alpha hydrogen.

SECTION – B (2 marks) (2*40=80)

61) c $y = \frac{2x}{3x+4}$

$3xy + 4y = 2x$

$3xy - 2x = 4y$

$x = \frac{4y}{3y-2}$

x is defined when $3y - 2 \neq 0$

i.e., $y = \frac{2}{3}$

Range $= R - \{2/3\}$

62) a $\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}$

$R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$

$\Delta = \begin{vmatrix} x+1 & x+2 & x+4 \\ 2 & 3 & 4 \\ 6 & 8 & 10 \end{vmatrix}$

$C_3 \rightarrow C_3 - C_2, C_2 \rightarrow C_2 - C_1$

$\begin{vmatrix} x+1 & 1 & 3 \\ 2 & 1 & 2 \\ 6 & 2 & 4 \end{vmatrix}$

$R_3 \rightarrow R_3 - 2R_2$

$\begin{vmatrix} x+1 & 1 & 3 \\ 2 & 1 & 2 \\ 2 & 0 & 0 \end{vmatrix} = 2\begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} = -2$

63) c According to question, $t_{20} = t_{21}$
$C(44, 20)\ x^{20} = C(44, 21)\ x^{21}$
$x = \frac{C(44,20)}{C(44,21)} = \frac{7}{8}$

64) d Let given expression = e^S
But, $S = \log\{1 + (1 + x)\} = \log(2 + x)$
$\therefore e^S = e^{\log(2+x)} = 2 + x$

65) c $\tan^2(\sec^{-1} 2) = \sec^2(\sec^{-1} 2) - 1 = [sec(\sec^{-1} 2)]^2 - 1 = 2^2 - 1 = 3$
$\cot^2(\text{cosec}^{-1} 3) = \text{cosec}^2(\text{cosec}^{-1} 3) - 1 = [cosec(\text{cosec}^{-1} 3)]^2 - 1 = 3^2 - 1 = 8$
$\tan^2(\sec^{-1} 2) + \cot^2(\text{cosec}^{-1} 3) = 3 + 8 = 11$

66) b Comparing $4x^2 + 2hxy - 7y^2 = 0$ with $ax^2 + 2hxy + by^2 = 0$
$a = 4, b = -7$
Let m_1 and m_2 be the slopes of given lines.
Sum of slope $(m_1 + m_2) = -\frac{2h}{b} = \frac{2h}{7}$
Product of slopes $(m_1m_2) = \frac{a}{b} = -\frac{4}{7}$
As, $m_1 + m_2 = m_1m_2$
$\frac{2h}{7} = -\frac{4}{7}$
$h = -2$

67) d $S_1 : x^2 + y^2 = 16$
Centre $(c_1) = (0, 0)$ and $r_1 = 4$
$S_2 : x^2 + y^2 - 2y = 0$
Centre $(c_2) = (0, 1)$ and $r_2 = 1$
$\therefore c_1c_2 = \sqrt{(0-0)^2 + (1-0)^2} = 1$
$r_1 - r_2 = 4 - 1 = 3$
$\Rightarrow c_1c_2 < (r_1 - r_2)$
So, they do not touch each other and lies completely within each other. Thus, there is no real common tangent to the given circle.

68) b Equation of the hyperbola is:
$\frac{x^2}{1/2} - \frac{y^2}{1/2} = 1$
${e_1}^2 = \frac{a^2+b^2}{a^2}$
$e_1 = \sqrt{2}$
Equation of the ellipse is:
$\frac{x^2}{2} + \frac{y^2}{1} = 1$
${e_2}^2 = \frac{a^2-b^2}{a^2} = \frac{2-1}{2} = \frac{1}{2}$
$e_2 = \frac{1}{\sqrt{2}}$
So, $e_1e_2 = 1$

69) a $\alpha = 45^\circ, \beta = 60^\circ, \gamma = ?$
We have, $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$
$cos^2 45^\circ + cos^2 60^\circ + cos^2\gamma = 1$
$cos^2\gamma = \frac{1}{4}$
$\cos\gamma = \pm\frac{1}{2}$
$\gamma = 60^\circ$ or 120°

70) b Here, $\vec{a} = (4, 3)$ and $\vec{b} = (-2, -3)$

Then, $\vec{a} + 2\vec{b} = (4, 3) + (-4, -6) = (0, -3)$

And the unit vector of $\vec{a} + 2\vec{b} = \frac{\vec{a}+2\vec{b}}{|\vec{a}+2\vec{b}|} = \frac{(0,-3)}{\sqrt{0^2+(-3)^2}} = (0, -1)$

71) b Mode – Mean = 12
We have, Mode = 3 Median – 2 Mean
Mode – Mean = 3 (Median – Mean)
12 = 3 (Median – Mean)
Median – Mean = 4
Again, Mode = 3 Median – 2 Mean
Mode – Median = 2(Median – Mean) = 2 × 4 = 8

72) b $\lim_{x\to 0} \frac{a^x - b^x}{x} = \lim_{x\to 0} \frac{(a^x-1)-(b^x-1)}{x} = \lim_{x\to 0} \frac{a^x-1}{x} - \lim_{x\to 0} \frac{b^x-1}{x} = \log a - \log b = \log \frac{a}{b}$

73) d $\frac{d}{dx}\left[f(x)^{g(x)}\right] = f(x)^{g(x)} \frac{d\, g(x) \log f(x)}{dx}$

$\frac{d}{dx} x^x = x^x \frac{d\, x \log x}{dx} = x^x \left[\log x + x.\frac{1}{x}\right] = x^x[\log x + 1]$

74) c $I = \int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

Let, $\sin^{-1} x = y$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

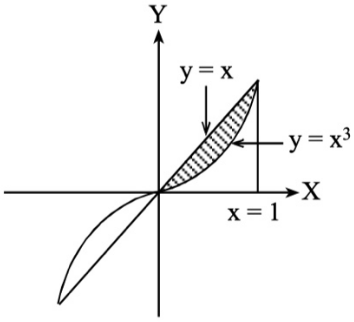
$dy = \frac{dx}{\sqrt{1-x^2}}$

So, $I = \int \sin y \,.\, y.\, dy = y \int \sin y\, dy + \int \left[\frac{dy}{dx}. \int \sin y\, dy\right] dy = y \cos y + \sin y + c$

$= -\sin^{-1} x \sqrt{1-x^2} + x + c$

$= x - \sqrt{1-x^2} \sin^{-1} x + c$

75) a Solving $y = x^3$ and $y = x$, $x = \pm 1$
Area of region in first quadrant



$A = \int_0^1 (y_1 - y_2) dx = \int_0^1 (x - x^3) dx = \left|\frac{x^2}{2} - \frac{x^4}{4}\right|_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ sq. unit

76) a Using $v = v_0 + at$

$t = \frac{v_0}{a} = \frac{30}{10} = 3$ s

Also, $x_n = v_0 + \frac{a}{2}(2n - 1) = 5$ m

77) b Use law of conservation of energy

$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$

$\frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = mg\left(\frac{3v^2}{4g}\right)$ $\quad \because (v = R\omega)$

$\frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = mg\left(\frac{3v^2}{4g}\right)$

$\frac{1}{2}I\left(\frac{v}{R}\right)^2 = \frac{3}{4}mv^2 - \frac{1}{2}mv^2 = \frac{1}{4}mv^2$

$I = \frac{1}{4}mv^2 \times \frac{2R^2}{v^2} = \frac{1}{2}mR^2$ (disc)

78) d Using, $g_d = g\left(1 - \frac{d}{R}\right)$

Multiplying by 'm' on both sides, we get,

$mg' = mg\left(1 - \frac{d}{R}\right)$ where, $d = \frac{R}{2}$

$mg' = 200\left(1 - \frac{R}{2R}\right) = \frac{200}{2} = 100$ N

79) a Let the side of the cube be L.

Then volume of the cube outside = volume of water displaced due to mass

The water displaced is 300 g and volume is 300 cm^3.

Hence, $3 \times L \times L = 300 \Rightarrow L = 10$ cm

80) c Heat gained by ice = mL = 20 × 80 = 1600

Heat lost by water = 20 t

By principle of calorimetry, Heat lost = Heat gained

$1600 = 20\,t$

$t = 80\,^{\circ}C$

81) b $\eta = \frac{\text{Work done by engine}}{\text{Heat supplied by source}}$

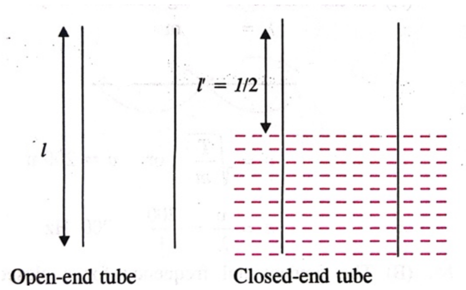
$1 - \frac{T_2}{T_1} = \frac{W}{40}$

$1 - \frac{300}{400} = \frac{W}{40}$

$\frac{1}{4} = \frac{W}{40}$

W = 10 kJ

82) d

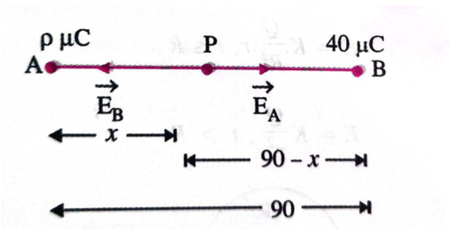


Fundamental frequency of an open end tube, $v_0 = \frac{v}{2l} = n$

When half of the open end tube is dipped in water, $l' = \frac{l}{2}$

Fundamental freuency of the closed end tube is, $v_c = \frac{v}{4l'} = \frac{v}{4\left(\frac{l}{2}\right)} = \frac{v}{2l} = v_0 = n$

83) d



Let at a distance x from point A, the electric field is zero.

Electric field at P due to charge A is:

$E_A = K\frac{q_A}{x^2} = \frac{K \times 10 \times 10^{-6}}{x^2}$

Similarly, $E_B = K\frac{q_B}{(90-x)^2} = \frac{K\times40\times10^{-6}}{(90-x)^2}$
Field at P will be zero if $E_A = E_B$
$\frac{K\times10\times10^{-6}}{x^2} = \frac{K\times40\times10^{-6}}{(90-x)^2}$
On solving, $x = 30$ cm

84) a Current in the circuit, $I = \frac{2}{16} = 0.125$ A
Potential drop across the wire, $V = IR = 0.125 \times 8 = 1$ V
Potential gradient $= \frac{1}{4} = 0.25\ \text{Vm}^{-1}$

85) b Here, $Z = \sqrt{R^2 + (X_L - X_C)^2} = R$
Hence, $I = \frac{100}{10} = 10$ A

86) c Apparent depth in the slab $= \frac{3}{1.5} = 2$ cm $\because \left(\mu = \frac{\text{Real depth}}{\text{Apparent depth}}\right)$
Total distance $= 2 + 2 = 4$ cm

87) a $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi = 4I + I + 2 \times 2I\cos\frac{\pi}{2} = 5I$

88) b $K.E. = \frac{hc[\lambda_0-\lambda]}{\lambda\lambda_0} = \frac{6.6\times10^{-34}\times3\times10^{8}[600-450]\times10^{-9}}{660\times450\times10^{-18}} = 0.625$ eV

89) b Volume of 1 mole of gas at STP is 22.4 L or 22400 mL or 22400 cc.
22400 cc of gas contains 6.023×10^{23} molecules.
1.12×10^{-7} cc contains $\frac{6.023\times10^{23}}{22400} \times 1.12 \times 10^{-7}$ molecules $= 3.01 \times 10^{12}$ molecules

90) c Normality = Molarity × n-factor
Normality $= \frac{\text{Weight of oxalic acid}}{\text{Molecular weight} \times \text{Volume (in litre)}} \times \text{n} - \text{factor}$
n-factor of oxalic acid = 2
Weight of oxalic acid $= \frac{1\times126\times1}{20\times2} = \frac{63}{20}$ gm

91) c Rate = k[R]
$1.5 \times 10^{-2} = k(0.5)$
$k = 3 \times 10^{-2}$ min^{-1}
Half life = $t_{1/2} = \frac{0.693}{k}$
$t_{1/2} = \frac{0.693}{0.03} = 23.1$ minutes

92) b At anode : $Fe^{2+} + 2e^- \rightarrow Fe$; $E^0 = -0.441$ V
At cathode : $2Fe^{3+} + 2e^- \rightarrow 2Fe^{2+}$; $E^0 = 0.771$ V
$E^0{}_{cell} = E^0{}_{cathode} - E^0{}_{anode} = 0.771 - (-0.441) = 1.212$ V

93) c $NH_4Cl + NaNO_3 \rightarrow NH_4NO_3$
$NH_4NO_3 \rightarrow N_2O + H_2O$

94) a $Cl_2 + NaOH$ (hot and conc.) $\rightarrow NaCl + NaClO_3 + H_2O$
$Cl_2 + NaOH$ (cold) $\rightarrow NaCl + NaClO + H_2O$

95) c

96) b $C_6H_5NO_2 \xrightarrow{Fe/HCl} C_6H_5NH_2 \xrightarrow{NaNO_2+HCl,273\ K} C_6H_5N \equiv NCl \xrightarrow{H_2O,283\ K} C_6H_5OH$
Nitrobenzene Aniline Benzene diazonium chloride Phenol

97) d
98) c
99) d
100) a

❖❖❖❖ Thank You!!! ❖❖❖❖