INSTITUTE OF ENGINEERING

MODEL ENTRANCE EXAM

(Beats Test Series – Day 3)
Solutions

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Date : 2081/05/04
(August 20)

Duration : 2 hours
Time : 8 A.M. – 10 A.M.

SECTION – A (1 marks) (1*60 = 60)

1) a
2) b
3) d
4) b
5) d
6) a
7) a
8) a
9) b
10) a
11) b
12) b
13) d $240 = 2 \times 3 \times 5 \times 8$
So, $A = \{2, 3, 5\}, B = \{5, 7, 8\}$
Clearly, $8 \in A \cup B$
14) d We have, $\text{adj}(AB) = \text{adj}(B)\,\text{adj}(A)$

15) b $2e^{i\theta} = 2(\cos\theta + i\sin\theta)$
$2e^{i\pi/3} = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 2\left(\frac{1+\sqrt{3}i}{2}\right) = 1 + \sqrt{3}i$
16) c $S_n = 5 + 10 + 15 + \cdots + 100$
$a = 5, n = 20$
$S_n = \frac{n}{2}(a + l) = \frac{20}{2}(5 + 100) = 1050$
17) d Here, the first person can occupy any of the five seats. So, there are 5 ways in which the first person can seat himself. Again, the second person can occupy any of the remaining 4 seats. So, he can be seated in 4 ways. Similarly, the third person can occupy a seat in 3 ways. The required number of ways $= 5 \times 4 \times 3 = 60$.
18) c Sum of odd binomial coefficient $= 2^{n-1}$
i.e., $C_1 + C_3 + C_5 + C_7 + C_9 = 2^{10-1} = 2^9$
19) c $7sin^2x + 3cos^2x = 4$
$7sin^2x + 3 - 3sin^2x = 4$
$4sin^2x = 1$
$\sin x = \left(\frac{1}{2}\right)^2$
$\sin x = \left(\sin\frac{\pi}{6}\right)^2$
$x = n\pi \pm \frac{\pi}{6}$
20) c $\text{cosec}^{-1}\left\{\text{cosec}\frac{5\pi}{4}\right\} = \text{cosec}^{-1}\left\{\text{cosec}\left(\pi + \frac{\pi}{4}\right)\right\} = \text{cosec}^{-1}\left\{-\text{cosec}\frac{\pi}{4}\right\} = -\frac{\pi}{4}$
21) b $\frac{\cos B}{b} + \frac{\cos C + \cos A}{c+a} = \frac{b\cos C + b\cos A + c\cos B + a\cos B}{(c+a)b} = \frac{(b\cos C + c\cos B) + (b\cos A + a\cos B)}{(c+a)b}$
$= \frac{a+c}{(c+a)b} = \frac{1}{b}$
22) b Centroid divides the join of orthocentre and circum-centre in the ratio 2 : 1 i.e.
By using section formula we get;
$x = \frac{2.6 + 1(-3)}{2+1} = 3$

$y = \frac{2.2+1.5}{2+1} = 3$
∴ Centroid is (3, 3).

23) c Here, the equation is:
$x^2 = 12y = 4.3y$
Comparing $x^2 = 4ay$, $a = 3$
Length of latus rectum $= 4a = 4 \times 3 = 12$

24) c Asymptotes are:
3x + 4y = 2
4x – 3y = 5
Product of their slopes is $\left(-\frac{3}{4}\right)\left(\frac{4}{3}\right) = -1$
Asymptotes are perpendicular to each other. Hence, the hyperbola is a rectangular hyperbola.
Eccentricity (e) $= \sqrt{2}$

25) a Required distance is:
$\left|\frac{\frac{5}{2}+8}{\sqrt{4+1+4}}\right| = \frac{7}{2}$

26) b $\vec{a} = (\hat{\imath} + \hat{\jmath} + \hat{k}), \vec{b} = (\hat{\imath} - \hat{\jmath} + \hat{k})$
Now, $\vec{a}.\vec{b} = 1 - 1 + 1 = 1$
Also, $|\vec{b}| = \sqrt{3}$
Now, projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{1}{\sqrt{3}}$

27) c $n = 4, p = \frac{1}{2}, q = \frac{1}{2}, r = 2$
$P(r) = C(4,2) \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^2 = \frac{4\times3\times1}{2} \times \frac{1}{4\times4} = \frac{3}{8}$

28) c $\lim_{x\to\infty}\left(1 + \frac{5}{x}\right)^x = e^5$
$\left[\lim_{x\to\infty}\left(1 + \frac{m}{x}\right)^x = e^m\right]$

29) d $\frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{\frac{d}{dx}\tan^{-1}x}{\frac{d}{dx}\cot^{-1}x} = \frac{\frac{1}{1+x^2}}{-\frac{1}{1+x^2}} = -1$

30) d $f(x) = x^3 + 3x^2 - 9x + 12$
$f'(x) = 3x^2 + 6x - 9$
$f''(x) = 6x + 6$
For point of inflection,
$f''(x) = 0$
$6x + 6 = 0$
$x = -1$

31) b $\int_0^\pi cos^3x\,dx = \frac{1}{4}\int_0^\pi(\cos 3x + 3\cos x)\,dx \quad [\because \cos 3x = 4cos^3x - 3\cos x]$
$= \frac{1}{4}\left[\frac{\sin 3x}{3} + 3\sin x\right]_0^\pi = 0$

32) a $x\frac{dy}{dx} + 2y = x^2 \ln x$
Dividing by x on both sides,
$\frac{dy}{dx} + \frac{2}{x}.y = x\ln x$
Here, $p = \frac{1}{2}$
So, I.F. $e^{\int\frac{2}{x}dx} = e^{2\ln x} = e^{\ln x^2} = x^2$

33) b

34) c $\frac{u^2}{2g} = h$

Now, $R_{max} = \frac{u^2 \sin 90^\circ}{g} = \frac{u^2}{g} = 2h$

35) a Acceleration. $a = \omega^2 r = \frac{v^2}{r} = \omega v = \frac{2\pi}{T} v$

36) c Total energy $= \frac{1}{2} m\omega^2 a^2 = constant$

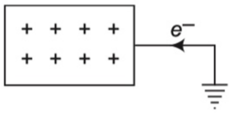
37) b $4\pi r_1{}^2 T_1{}^4 = 4\pi r_2{}^2 T_2{}^4$

$\frac{r_1}{r_2} = \frac{T_2{}^2}{T_1{}^2} = \left(\frac{T_2}{T_1}\right)^2$

38) c For a given pressure, volume will be more if temperature is more (Charle's law).
From the graph, it is clear that: $V_2 > V_1 \Rightarrow T_2 > T_1$.

39) b

40) a



When positively charged body connected to earth, electrons flow from earth to body and body becomes neutral.

41) d Work done by magnetic force is zero.

42) b

43) a Image formed is real, inverted and same in size because object is at the centre of curvature of the mirror.

44) b

45) b To make a p-type semiconductor trivalent impurity is added, boron is trivalent.

46) c

47) c Cu and O combines to form two compound CuO and Cu_2O.

48) c For p-orbital, $l = 1$

Orbital angular momentum $= \sqrt{l(l+1)}\frac{h}{2\pi} = \sqrt{1(1+1)}\frac{h}{2\pi} = \frac{h}{\sqrt{2}\pi}$

49) d $\frac{r_{H_2}}{r_{O_2}} = \sqrt{\frac{16}{2}} = \sqrt{8}$, which is maximum difference in rate out of given pair.

50) d Enthalpy, Internal energy, Volume are all extensive as they depends on amount of substances.

51) d $k_p = \frac{p_{N_2} \times p^3{}_{H_2}}{p^2{}_{NH_3}} = \frac{(atm)^4}{(atm)^2} = (atm)^2$

52) b $NH_2{}^- + H^+ \rightarrow NH_3$

53) b LiCl is covalent compound, so it is soluble in organic solvents.

54) d

55) d Graphite has free electrons. So, it is a good conductor.

56) d

57) d Hydro metallurgy involves both leaching and precipitation of the metals from its solution by adding precipitating agent.

58) c Glycerol decompose below its boiling point. So, it is purified by vacuum distillation or distillation under reduced pressure.

59) a Methanal does not have $\alpha - H$. So, it does not shows tautomers.

60) a Lower the electronegativity of the carbon bearing +ve charge, more stable is the carbocation. Electronegativity follows the order : $sp > sp^2 > sp^3$.

SECTION – B (2 marks) (2*40=80)

61) d We know that the domain of $\sin^{-1} x$ is [-1, 1].
$\therefore 1 \le \log_3\left(\frac{x}{3}\right) \le 1$
$3^{-1} \le \frac{x}{3} \le 3^1$
$1 \le x \le 9$
i.e., $x \in [1, 9]$

62) b $D_1 = (x^3 + ab^2 + a^2b) - (abx + abx + abx) = x^3 - 3abx + a^2b + ab^2$
$\frac{d}{dx}(D_1) = 3x^2 - 3ab = 3(x^2 - ab)$
Again, $D_2 = x^2 - ab$
Thus, $\frac{d}{dx}(D_1) = 3D_2$

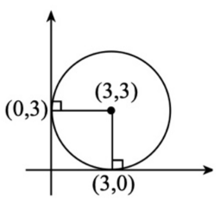
63) b Let the roots of the equation be 3α and 4α.
Sum of the roots $= 3\alpha + 4\alpha = -\frac{q}{p}$
$\alpha = -\frac{q}{7p}$
Product of the roots $3\alpha . 4\alpha = \frac{r}{p}$
$12\alpha^2 = \frac{r}{p}$
$12\left(-\frac{q}{7p}\right)^2 = \frac{r}{p}$
$12q^2 = 49pr$

64) a $\frac{1}{2}.\frac{1}{3} - \frac{1}{4}.\frac{1}{3^2} + \frac{1}{6}.\frac{1}{3^3} + \cdots + \infty = \frac{1}{2}\left[\frac{\left(\frac{1}{3}\right)}{1} + \frac{\left(\frac{1}{3}\right)^2}{2} + \frac{\left(\frac{1}{3}\right)^3}{3} + \cdots + \infty\right] = \frac{1}{2}\log\left(1 + \frac{1}{3}\right) = \frac{1}{2}\log\left(\frac{4}{3}\right)$

65) b tan2x tanx = 1
$\frac{2\tan x}{1 - tan^2 x}.\tan x = 1$
$2tan^2 x = 1 - tan^2 x$
$3tan^2 x = 1$
$\tan x = \pm\frac{1}{\sqrt{3}} = \tan\left(\pm\frac{\pi}{6}\right)$
$x = n\pi \pm \frac{\pi}{6}, n \in 1$
$= (6n \pm 1)\frac{\pi}{6}$

66) c Comparing with $ax^2 + 2hxy + by^2 = 0$
$a = 1, b = 1, h = \operatorname{cosec}\theta$
Then, the bisectors are given by: $h(x^2 - y^2) = (a - b)xy$
$\operatorname{cosec}\theta\,(x^2 - y^2) = (1 - 1)xy$
$x^2 - y^2 = 0$
$\therefore (x - y)(x + y) = 0$

67) d



Centre (h, k) = (3, 3)
Circle touching both the axes: r = h = k = 3

Equation of circle is: $(x-h)^2+(y-k)^2=r^2$
or, $(x-3)^2+(y-3)^2=3^2$
$\therefore x^2+y^2-6x-6y+9=0$

68) a Let the equation of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
Where, $2a=10 \Rightarrow a=5$
$ae=2 \Rightarrow e=\frac{2}{5}$
$b^2=a^2(1-e^2)=25\left(1-\frac{4}{25}\right)=21$
Required equation of the ellipse is:
$\frac{x^2}{25}+\frac{y^2}{21}=1$

69) a $AB=\sqrt{(-2+1)^2+(4-2)^2+(3-5)^2}=3$
Dc's of AB are:
$\frac{x_2-x_1}{AB},\frac{y_2-y_1}{AB},\frac{z_2-z_1}{AB}$
$=-\frac{1}{3},\frac{2}{3},-\frac{2}{3}$

70) b $\vec{a}-\vec{b}+\vec{c}=0$
$\vec{a}+\vec{c}=\vec{b}$
On squaring; $(\vec{a}+\vec{c})^2=\left(\vec{b}\right)^2$
$2\vec{a}.\vec{c}=b^2-a^2-c^2$
$2\vec{c}.\vec{a}=1-1-1$
$\vec{c}.\vec{a}=-\frac{1}{2}$

71) b Arranging the observations in ascending order: 150, 210, 340, 300, 310, 320, 340.
Clearly, the middle observation is 300.
$\therefore$ Median $=300$

Calculation of Mean Deviation

x_i	$\lvert d_i\rvert = \lvert x_i - 300\rvert$
340	40
150	150
210	90
240	60
300	0
310	10
320	20
Total	$d_i = \Sigma\lvert x_i - 300\rvert = 370$

$\therefore$ M.D. $=\frac{1}{n}\sum|d_i|=\frac{1}{7}\sum|x_i-300|=\frac{370}{7}=52.8$

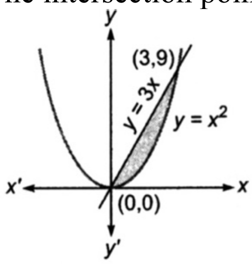
72) b $\lim\limits_{x\to 1}\frac{x+x^2+x^3+\cdots+x^n-n}{x-1}$
By L-Hospital's rule:
$=\lim\limits_{x\to 1}\frac{1+2x+3x^2+\cdots+nx^{n-1}-0}{1}$
$=1+2+3+\cdots+n$
$=\frac{n(n+1)}{2}$

73) c $y=\sqrt{\tan x+y}$
Squaring: $y^2=\tan x+y$

$2y.\frac{dy}{dx} = sec^2x + \frac{dy}{dx}$
$\frac{dy}{dx} = \frac{sec^2x}{(2y-1)}$

74) b $I = \int \frac{dx}{(x+3)\sqrt{x+2}}$
Put $\sqrt{x+2} = z$
$x + 2 = z^2$
$\frac{1}{2\sqrt{x+2}}dx = dz$
Now, $I = \int \frac{dx}{\sqrt{x+2}}.\frac{1}{x+3} = \int \frac{2\,dz}{(z^2-2+3)} = \int \frac{2\,dz}{z^2+1} = 2\tan^{-1}z + c$
$= 2\tan^{-1}\sqrt{x+2} + c$

75) c The intersection points of given curves are (0, 0) and (3, 9).



∴ Required area $= \int_0^3 (3x - x^2)\,dx = \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 = \frac{27}{6} = 4.5$ sq units

76) a $\frac{u}{4} = u - at_0$
$a = \frac{3u}{4t_0}$
$\frac{u}{a} = \frac{4}{3}t_0$
Now, $0 = u - at$
$t = \frac{u}{a} = \frac{4}{3}t_0$

77) a Force exerted by man on rope transfers to it in the form of tension.
Net upwards force on the system is 2T or 2F.
Net downward force is(50 + 30) g = 8 g.
For equilibrium of system, 2F = 80 g or F = 40g

78) d $\pi R = L$
$R = \frac{L}{\pi}$
$V = -\frac{GM}{R} = -\frac{\pi GM}{L}$

79) d $V = \frac{\pi\rho r^4}{8\eta l}$
$V \propto \rho r^4$
$\frac{V_2}{V_1} = \left(\frac{\rho_2}{\rho_1}\right)\left(\frac{r_2}{r_1}\right)^4 = 2 \times \left(\frac{1}{2}\right)^4 = \frac{1}{8}$
$V_2 = \frac{Q}{8}$

80) c $ms\Delta\theta = \frac{1}{2}\left(\frac{1}{2}mv^2\right)$
$\therefore \Delta\theta = \frac{v^2}{4s} = \frac{(300)^2}{4\times150} = 150\,^\circ C$

81) d $P_1{V_1}^\gamma = P_2{V_2}^\gamma$
$\frac{P_2}{P_1} = \left[\frac{V_1}{V_2}\right]^\gamma = \left[\frac{4}{1}\right]^{3/2} = \frac{8}{1}$

82) d Frequency of sound does not change with medium, because it is characteristic of source.

83) b $V = \frac{q}{C}$ or $V \propto \frac{1}{C}$

V has reduced to $\frac{1}{8}^{\text{th}}$ of its original value. Therefore, C has increased 8 times.

84) c Current divides according to resistance, so current in 6Ω 0.8 resistance is $\frac{0.8}{2} = 0.4$ A

So, total current in circuit is0.8 + 0.4 = 1.2A

∴ Potential drop across 4Ω = 1.2 × 4 = 4.8V

85) d Work done in rotating the magnet from angle θ_1 to θ_2 is given by:

$W = MB(\cos\theta_1 - \cos\theta_2) = MB(\cos 0^\circ - \cos 180^\circ) = 2MB$

86) c Light should fall normally on the silvered face.

$r_2 = 0$

$r_1 = A = 30^\circ$

Now, $\mu = \frac{\sin i_1}{\sin r_1}$

$\sqrt{2} = \frac{\sin i_1}{\sin 30^\circ}$

$i_1 = 45^\circ$

87) d Distance of n^{th} bright fringe from the centre,

$y_n = \frac{nD\lambda}{d}$

$y_n = \frac{3 \times 6000 \times 10^{-10} \times 25}{0.5 \times 10^{-3}} = 9 \times 10^{-3}$ m = 9 mm

88) c Work function, $W_0 = \frac{hc}{\lambda_0}$

$\frac{(W_0)_T}{(W_0)_{Na}} = \frac{\lambda_{Na}}{\lambda_T}$

$\lambda_T = \frac{\lambda_{Na} \times (W_0)_{Na}}{(W_0)_T} = \frac{5600 \times 2.3}{4.5} = 2862$ Å

89) b $CH_3COOK \xrightarrow{Electrolysis} CH_3 - CH_3 + K_2CO_3 + H_2$

90) c $R - C \equiv N + [H] \xrightarrow{Na/C_2H_5OH} RCH_2NH_2$

91) c In BF_3 and $\overset{-}{NO_2}$, central atom have sp^2 hybridization and in $\overset{-}{NH_2}$ and H_2O, central atom have sp^3 hybridization.

92) b $Cu^{2+} + 2e^- \rightarrow Cu$

One Faraday deposits Cu = 1/2 moles

$= \frac{1}{2} \times 6.02 \times 10^{23}$ atoms

$= 3.01 \times 10^{23}$ atoms

93) b For initial concentration to reduce to $\frac{1}{8}^{th}$, number of half life = 3.

So, time required = 13 × 3 = 42 s

94) a Fe_3O_4 is composed of Fe_2O_3 and FeO. So, the oxidation states are +3 and +2.

95) c Mn_2O_7 is acidic, V_2O_5 is amphoteric and CrO is basic.

96) d In contact process, SO_3 is dissolved in H_2SO_4 forming $H_2S_2O_7$ (oleum).

97) b

98) b

99) b

100) b

❖❖❖❖ Thank You!!! ❖❖❖❖