BEATS ENGINEERING

INSTITUTE OF ENGINEERING

MODEL ENTRANCE EXAM

(Beats Test Series – Day 4)
Solutions

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Date : 2081/05/05
(August 21)

Duration : 2 hours
Time : 8 A.M. – 10 A.M.

SECTION – A (1 marks) (1*60 = 60)

1) b
2) a
3) d
4) b
5) d
6) c
7) d
8) c
9) b
10) a
11) a
12) b
13) b $\lim_{x\to1}(1-x)\tan\frac{\pi x}{2} = \lim_{x\to1}\frac{1-x}{\cot\frac{\pi x}{2}}\left(\frac{0}{0}\text{ form}\right)$

$= \lim_{x\to1}\frac{-1}{-\frac{\pi}{2}\operatorname{cosec}^2\frac{\pi x}{2}} = \frac{-1}{-\frac{\pi}{2}\times1} = \frac{2}{\pi}$

14) d $\frac{d}{dx}\left(\frac{\cos x}{\sin x+1}\right) = \frac{(\sin x+1).(-\sin x)-\cos x.\cos x}{(\sin x+1)^2} = \frac{-sin^2x-\sin x-cos^2x}{(\sin x+1)^2} = \frac{-(sinx+1)}{(\sin x+1)^2} = \frac{-1}{\sin x+1}$

15) d $xy = 4 \Rightarrow y = \frac{4}{x}$

$x + 16y = x + \frac{64}{x} = x + \frac{8^2}{x}$

Note:

The function $f(x) = x + \frac{a^2}{x}$ $(a > 0)$ has:

i) local maxima at $x = -a$ and

ii) local minima at $x = +a$

Here, maxima occur at $x = -8$.

Hence, maximum value $= -8 - \frac{64}{8} = -16$

16) c $\int\frac{dx}{x+\sqrt{x}} = \frac{dx}{\sqrt{x}(\sqrt{x}+1)} = 2\int\frac{\frac{1}{2\sqrt{x}}dx}{\sqrt{x}+1} = 2\log(\sqrt{x}+1) + c$

17) c Let us assume roots as 2 and 3.

$x^2 - (2+3)x + 2\times3 = 0$

$x^2 - 5x + 6 = 0$

Comparing it with the given equation, we get: $k = 5$.

18) c $\sum_{n=0}^{\infty}\frac{(log_ex)^{2n}}{(2n)!} = 1 + \frac{(log_ex)^2}{2!} + \frac{(log_ex)^4}{4!} + \cdots + \infty = \frac{e^{\log x}+e^{-\log x}}{2} = \frac{x+x^{-1}}{2}$

19) c Let r be the common ratio

$405 = 5r^4$

$r^4 = 81$

$r = 3$

$\therefore z = \frac{405}{3} = 135$

20) b Total number of balls = 30
Number of boundaries the batsman hit = 6
Number of balls without boundaries = 24
So, the probability when there is no boundary is: 24/30 = 4/5

21) b $A - B$ exists when A and B are of the same order. So, $p = r, q = s$.

22) b He fails if he fails in at least one of given 5 subjects.
So, total number of ways $= 2^5 - 1 = 31$

23) d $fog(x) = f(g(x)) = f\left(\frac{x}{1-x}\right) = \frac{\frac{x}{1-x}}{\frac{x}{1-x}+1} = \frac{x}{x+1-x} = x$

24) a Adding given equations, we get:
$(a + b + c)x + (a + b + c)y + (a + b + c) = 0$
$0.x + 0.y + o = 0 \quad (\because a + b + c = 0)$
Thus, lines are concurrent.

25) c Given equations can be written as:
$x^2 + 2(2)xy + ky^2 = 0$
It represents coincident lines if $h^2 - ab = 0$
$2^2 - 1 \times k = 0$
$k = 4$

26) b The line $y = mx + c$ intersects the parabola $y^2 = 4ax$ in
i) two real points if $\frac{mc}{a} < 1$ (chord)
ii) one single point if $\frac{mc}{a} = 1$ (tangent)
iii) two imaginary points if $\frac{mc}{a} > 1$ (outside)

27) a $16x^2 + 25y^2 = 400$
$\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$
Here, $a > b$
So, vertices $= (\pm a, 0) = (\pm 5, 0)$

28) d Equation of plane passing through the point (x_1, y_1, z_1) and making equal intercepts on the coordinate axes is:
$x + y + z = x_1 + y_1 + z_1$
Here, $x + y + z = 1 - 1 + 2$
$\therefore x + y + z = 2$

29) d $4 \sin A \, cos^3 A - 4 \cos A \, sin^3 A = 4 \cos A \sin A \, (cos^2 A - sin^2 A) = 2(2 \sin A \cos A)(\cos 2A)$
$= 2(\sin 2A)(\cos 2A) = \sin 4A$

30) d $sin^2\theta + 3 \cos\theta = 3$
$1 - cos^2\theta + 3\cos\theta = 3$
$cos^2\theta - 3\cos\theta + 2 = 0$
$(\cos\theta - 1)(\cos\theta - 2) = 0$
$\cos\theta = 1, \cos\theta = 2$ (not possible)
$\theta = 0$ in $[-\pi, \pi]$
Thus, there is only one solution.

31) a For $x = -\frac{1}{2}$, $\sin^{-1}\left(\frac{1}{x}\right) = \sin^{-1}(2)$, which is not defined.

32) c $\vec{a}.\vec{b} \geq 0$
$ab\cos\theta \geq 0$
$\cos\theta \geq 0$
$0 \leq \theta \leq \frac{\pi}{2}$

33) c Number of molecules in 14 g of N_2 is same as the number of molecules in 16 g of O_2.

34) b For $l = 2, m = -3$ is not possible.

35) c HCl is polar whereas He is non polar. So, they have dipole induced dipole interaction.

36) b 1 mole of H_2SO_4 = 2 moles of H^+
2 moles of NaOH = 2 moles of OH^-
1 moles of H^+ ions on neutralization gives 57.3 kJ.
2 moles of H^+ ions on neutralization gives 57.3 × 2 kJ.

37) c Equilibrium constant changes with change in temperatures.

38) a $Ag_2CrO_4 \rightleftharpoons 2Ag^+ + CrO_4{}^{2-}$
$K_{sp} = [Ag^+]^2[CrO_4{}^{2-}]$

39) a The given atomic number is of Phosphorous (P).
So, the atomic number of an element which is just below the above element in the periodic table = 15 + 18 =33

40) d Radioactive isotope of hydrogen is tritium ${}_1^3H$ which contains 1 proton and 2 neutrons.

41) c Lattice energy is directly proportional to charge and inversely proportional to size.

42) b

43) a C forms CO which is neutral and CO_2 which is acidic oxide.

44) b Towards ESR, reactivity order is:
$NH_2 > OH > OCH_3 > H > X > CHO > COOH > SO_3H > CN > NO_2$

45) c

46) a Acidic strength $\propto \frac{-I\,|-R}{+I\,|+R}$

47) a $[RC] = \left[\frac{V}{I} \times \frac{q}{V}\right] = \left[\frac{q}{I}\right] = \left[\frac{charge}{current}\right] = [time] = T$

48) b

49) a

50) d Rise of liquid in a capillary tube is due to surface tension.

51) c Water cools to 0˚C so ice and water are at the same temperature. Water cannot give out its latent heat to ice and cannot freeze.

52) d

53) c If incident angle of light beam is more than the critical angle, total internal reflection occurs.

54) b $P = 1/f$. A negative power indicates a concave lens. $f = -\frac{1}{2.5} = 0.4\,m$. So, the lens is concave with 0.4 m.

55) c

56) b A charged conductor has the same potential (V) at all points whatever its shape. So, along its surface $dV = 0, E = \frac{dV}{dr} = 0$, along its surface i.e., the component of E at a point on the surface is zero. Hence, E is normal to the surface at the point.

57) c In series, current remains same in each component.

58) d Resistance of semiconductor decreases with increase in temperature.

59) d

60) a Since, superconducting material and liquid nitrogen both are diamagnetic in nature, the dipped ball of superconductor in liquid nitrogen also behaves as a diamagnetic material. When it is placed near a magnet, it will be repelled.

SECTION – B (2 marks) (2*40=80)

61) d

62) a

63) d

64) d

65) d By sine formula

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{6}{\sin 30^\circ} = \frac{8}{\sin B}$

$\sin B = \frac{4}{6} = \frac{2}{3}$

$B = \sin^{-1}\frac{2}{3}$

$\sin^{-1} x = \sin^{-1}\frac{2}{3}$

$x = \frac{2}{3}$

66) a As given,

$sin^2\theta = \sin\phi . \cos\phi$

$2sin^2\theta = \sin 2\phi$

$1 - 2sin^2\theta = 1 - \sin 2\phi$

$\cos 2\theta = 1 - \cos\left(\frac{\pi}{2} - 2\phi\right)$

$\therefore \cos 2\theta = 2sin^2\left(\frac{\pi}{4} - \phi\right)$

67) b $\lim_{x\to\frac{\pi}{2}}\left[x\tan x - \left(\frac{\pi}{2}\right)\sec x\right] = \lim_{x\to\frac{\pi}{2}}\left[\frac{x\sin x}{\cos x} - \frac{\pi}{2\cos x}\right] = \lim_{x\to\frac{\pi}{2}}\left[\frac{x\sin x}{\cos x} - \frac{\pi}{2\cos x}\right] = \lim_{x\to\frac{\pi}{2}}\frac{2x\sin x-\pi}{2\cos x}$

$\left(\frac{0}{0}\text{ form}\right)$

$= \lim_{x\to\frac{\pi}{2}}\frac{[2\sin x+2x\cos x]}{-2\sin x}$ (Applying L'Hospital Rule)

$= \frac{2(1)+2.\frac{\pi}{2}.0}{-2(1)} = \frac{2}{-2} = -1$

68) b $f(x) = \frac{x-|x-1|}{x} = \begin{cases} \frac{x+x-1}{x}, & x < 1, x \neq 0 \\ \frac{x-(x-1)}{x}, & x \geq 1 \end{cases} = \begin{cases} \frac{2x-1}{x}, & x < 1, x \neq 0 \\ \frac{1}{x}, & x \geq 1 \end{cases}$

Clearly, $f(x)$ is discontinuous at $x = 0$ as it is not defined at $x = 0$. Since $f(x)$ is not defined at $x = 0$, $f(x)$ cannot be differentiable at $x = 0$.

And, $f(x)$ is continuous at $x = 1$, but it is not differentiable at $x = 1$, because $Lf'(1) = 1$ and $Rf'(1) = -1$.

69) d $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{x}(3-x)}{1-3x}\right)\right]$

Put $\sqrt{x} = \tan\theta \ or\ \theta = \tan^{-1} x$

$\frac{d}{dx}\left[\tan^{-1}\left(\frac{\tan\theta(3-tan^2\theta)}{1-3tan^2\theta}\right)\right] = \frac{d}{dx}\left[\tan^{-1}\left(\frac{3\tan\theta-tan^3\theta}{1-3tan^2\theta}\right)\right] = \frac{d}{dx}[\tan^{-1}(\tan 3\theta)] = \frac{d}{dx}(3\theta)$

$= \frac{d}{dx}(3\tan^{-1}\sqrt{x}) = 3\frac{d(\tan^{-1}\sqrt{x})}{\sqrt{x}}.\frac{d\sqrt{x}}{x} = 3.\frac{1}{1+x}.\frac{1}{2\sqrt{x}} = \frac{3}{2\sqrt{x}(1+x)}$

70) b $4x^2 + 9y^2 = 72$

Diff. w.r.t. x, we have

$8x + 18y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{4}{9}\frac{x}{y}$

At (3, 2), $\left(\frac{dy}{dx}\right)_1 = -\frac{4}{9}.\frac{3}{2} = -\frac{2}{3}$

Also, $x^2 - y^2 = 5$

Diff. w.r.t. x, we have

$2x - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x}{y}$

At (3, 2), $\left(\frac{dy}{dx}\right)_2 = \frac{3}{2}$

Since, $\left(\frac{dy}{dx}\right)_1 \times \left(\frac{dy}{dx}\right)_2 = -1$, the curves cut orthogonally.

71) b $\int_{\sqrt{2}}^{x}\frac{dx}{x\sqrt{x^2-1}} = \frac{\pi}{12}$

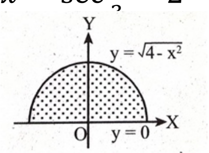
$|\sec^{-1}x|_{\sqrt{2}}^{x} = \frac{\pi}{12}$

$\sec^{-1}x - \frac{\pi}{4} = \frac{\pi}{12}$

$\sec^{-1}x = \frac{\pi}{3}$

$x = \sec\frac{\pi}{3} = 2$

72) a



When $y = 0, x = 2$ (radius)

Area of the semi-circle $= \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(2^2) = 2\pi$

73) b $T_{r+1} = {}^9C_r\, 3^{9-r}(ax)^r = {}^9C_r\, 3^{9-r}\, a^r x^r$

Coefficient of $x^r = {}^9C_r\, 3^{9-r}a^r$

As given,

Coefficient of x^2 = Coefficient of x^3

${}^9C_2\, 3^{9-2}a^2 = {}^9C_2\, 3^{9-3}a^3$

${}^9C_2\, 3^7a^2 = {}^9C_2\, 3^6a^3$

$a = \frac{9}{7}$

74) b Standard deviation is a measure of the spread of data points from the mean. Adding a constant to each value does not change the spread of the data, thus the standard deviation remains the same.

75) c $z = \frac{-2}{1+\sqrt{3}i} = \frac{-2}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i} = \frac{-2+2\sqrt{3}i}{1-3i^2} = \frac{-2+2\sqrt{3}i}{4} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$

$\therefore Arg(z) = \frac{2\pi}{3}$

76) b For real $f(x)$,

$5x - 3 - 2x^2 \geq 0$

$2x^2 - 5x + 3 \leq 0$

$(2x-3)(x-1) \leq 0$

$x \leq \frac{3}{2}$ and $x \geq 1$

$1 \leq x \leq \frac{3}{2}$

$\therefore D_f = \left[1, \frac{3}{2}\right]$

77) d Equation of circle is given by:

$(x+4)(x-12) + (y-3)(y+1) = 0$

$x^2 + y^2 - 8x - 2y - 51 = 0$

Length of y-intercept $= 2\sqrt{f^2 - c} = 2\sqrt{1+51} = 2\sqrt{52} = 4\sqrt{13}$

78) d Here, $e = \sqrt{2}$

Distance between directrices $= \frac{2a}{e} = 10$

$2a = 10\sqrt{2}$

Distance between foci $= 2ae = 10\sqrt{2} \times \sqrt{2} = 20$

79) c Dc's of the line making equal angles with the axes is:

$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Projection $= \frac{1}{\sqrt{3}}[(a-1) + (1+2) + (0-3)]$

$\frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}(a - 1 + 3 - 3)$

$2 = a - 1$

$a = 3$

80) a $KMnO_4 \rightarrow K_2MnO_4$. Here, only one electron change occurs.

+7 +6

So, equivalent weight = molecular weight.

81) b $E_{cell} = E^0_{cell} - \frac{0.0591}{2}\log\frac{[Mg^{2+}]}{[Sn^{2+}]}$

$= (2.34 - 0.14) - \frac{0.0591}{2}\log\frac{[10^{-2}]}{[10^{-1}]} = 2.23$ V

82) c 75% will involve 2 half-lives.

$t_{75\%} = 2 \times t_{1/2}$

$32 = 2 \times t_{1/2}$

$t_{1/2} = 16$ minutes

83) a % of N $= \frac{\text{mass of Nitrogen}}{\text{molecular weight of compound}} \times 100$

$20 = \frac{14}{M_w} \times 100$

$M_w = 70$

84) d V^{4+} (d^1 configuration is coloured). All other are colourless as they do not have unpaired electrons.

85) c Bleaching action of SO_2 is due to reduction and temporary and that of Cl_2 is due to oxidation and permanent.

86) d $CH \equiv CH \xrightarrow{\frac{HgSO_4}{H_2SO_4}, H_2O} CH_2 = CH - OH \xleftrightarrow{\text{Rearrangement}} CH_3CHO$

This reaction is known as Kucherov's reaction.

87) d It is aldol condensation reaction. So, β −hydroxy aldehyde is formed.

88) b If T be the Tension upward in vertical part of string, then for 6 kg block,

$mg - T = 0$

$6 \times 10 - T = 0$

$T = 60\ N$

Tension in horizontal part of string is also 60 N (System is in dynamic equilibrium)

Now for 10 kg block,

$T - f = 0$ (f = frictional force)

$60 - f = 0$

$f = 60\ N$

Hence, $\mu = \frac{f}{R} = \frac{60}{10g} = \frac{60}{100} = 0.6$

89) c $I'\omega' = I\omega$

$\left(\frac{MR^2}{2} + 2mR^2\right)\omega' = \frac{MR^2}{2}\omega$

$\left(\frac{M+4m}{2}\right)R^2\omega' = \frac{MR^2}{2}\omega$

$\therefore \omega' = \left(\frac{M}{M+4m}\right)\omega$

90) b $v_e = \sqrt{\frac{GM}{R}} \propto \sqrt{\frac{M}{R}}$

$v_e' = \sqrt{\frac{M_m R_e}{M_e R_m}} \times v_e = \frac{2}{9} \times 11 = 2.44\ km/s$

91) a For the particle executing SHM,

$v = \omega\sqrt{A^2 - x^2}$

$\frac{v^2}{\omega^2} = A^2 - x^2$

For $x = x_1, v = v_1$

$\frac{{v_1}^2}{\omega^2} = A^2 - {x_1}^2$ ------ (1)

For $x = x_2, v = v_2$

$\frac{{v_2}^2}{\omega^2} = A^2 - {x_2}^2$ ------ (2)

Subtracting (2) from (1),

$\frac{1}{\omega^2}({v_1}^2 - {v_2}^2) = {x_2}^2 - {x_1}^2$

$\omega^2 = \frac{{v_1}^2 - {v_2}^2}{{x_2}^2 - {x_1}^2}$

$\therefore \omega = \sqrt{\frac{{v_1}^2 - {v_2}^2}{{x_2}^2 - {x_1}^2}}$

92) c mass of steam × latent heat of vaporization = mass of ice × latent heat of fusion + $y \times s \times \Delta\theta$

$x \times 540 = y \times 80 + y \times 1 \times 100$

$\frac{x}{y} = \frac{180}{540}$

$\therefore \frac{x}{y} = \frac{1}{3}$

93) d Rate of heat flow, $R = \frac{Q}{t} = K\,A\frac{\Delta\theta}{l}$

$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)\left(\frac{l_2}{l_1}\right) = \left(\frac{r_1}{r_2}\right)^2\left(\frac{l_2}{l_1}\right) = \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right) = \frac{1}{8}$

94) a $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{\left(\frac{2}{3}R\right)} = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$

$\frac{3}{2R} = (\mu - 1).\frac{2}{R}$

$\frac{3}{4} = \mu - 1$

$\mu = \frac{7}{4} = 1.75$

95) b For the position of maxima,

$y_n = \frac{nD}{d}\lambda$

$x = \frac{3\times2}{0.2\times10^{-3}} \times 5 \times 10^{-7} = 1.5 \times 10^{-3}\,m = 1.5\,cm$

96) c $n' = \frac{v}{v - v_s}n = \frac{300}{300-200} \times 400 = 1200$ Hz

97) c For third charge q to be in equilibrium,

$\frac{1}{4\pi\varepsilon_0}\left[\frac{9qe}{x^2} - \frac{qe}{(a-x)^2}\right] = 0$

$\left(\frac{a-x}{x}\right)^2 = \frac{1}{9}$

$\frac{a-x}{x} = \frac{1}{3}$

$\therefore x = \frac{3}{4}a$

98) c $R = 3 + \frac{10\times(3+R)}{10+3+R} = 3 + \frac{30+10R}{13+R} = \frac{69+13R}{13+R}$

$13R + R^2 = 69 + 13R$

$R^2 = 69$

$\therefore R = \sqrt{69}\,\Omega$

99) a $I = \frac{V}{Z} = \frac{260}{\sqrt{50^2+120^2}} = \frac{260}{130} = 2$ A

100) b For $n = 3$ to $n = 2$ transition, $\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$

For $n = 4$ to $n = 2$ transition, $\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{16}$

$\frac{\lambda_2}{\lambda_1} = \frac{20}{27}$

$\lambda_2 = \frac{20}{27}\lambda_1 = \frac{20}{27}\lambda_0$

❖❖❖❖ Thank You!!! ❖❖❖❖